PROJECT COST-TIME OPTIMAL SOLUTIONS

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There have been several research works into the examination of the relationship between project total cost and total duration. To this end, the activity crashing technique stands out as an intricate, comprehensive and complex method for evaluating various cost-time scenarios. The technique can be used to identify the optimal solution: the project duration that yields the least project cost. However, what is regarded optimal for the contractor is likely to render a non-optimal solution for the client, and visa versa. Subsequently, depending on their priorities, the contractor and the client could examine a number of compromised solutions ranging between the 'least favourable solution' for the contractor, to the 'least favourable solution' to the client. Mathematical models are offered which represent the 'least compromise solution' and 'equal compromise solution'. In the case of 'least compromise solution', the total sum of compromises by both parties is minimised. Whereas, in the case of the 'equal compromise solution', both parties agree to compromise by an equal amount.

Keywords: activity crashing, optimal solution, project cost-time, mathematical modelling.

INTRODUCTION

For any given project, the relationship between the total cost and total duration (time) can be expressed as a curve which represents infinite possibilities of cost-time scenarios. The examination of the trade-off between project cost and duration can reveal the existence of a unique project duration that yields the lowest project cost. This particular combination is referred to as the *most economical solution*. However, the most economical solution for the contractor is likely to yield a non-economical solution for the client and vice-versa. This is due to the distinct differences between the cost-time trade-off curves for the contractor and for the client.

The most economical solution is a reflection of the contractor's (or the client's) cost-related priorities only. In other words, this solution is attractive only if the organisation's priority is to minimise the cost without due attention to project duration and other issues such as arrangements to accommodate risk sharing and the level of flexibility to deal with extensive variations. Cost minimisation is not always the most important criteria. Indeed, the generalisation of priorities, as a policy, undermines the intricate nature of any business operation. This is particularly true about construction projects, because they are envisaged as the building blocks of the firm's corporate objectives. In this respect, they are the means to an end rather than the end in themselves. Therefore the project-related priorities of each party are determined by their particular circumstances. Subsequently, for any particular project the possibility exists for the client and the contractor to negotiate a compromise solution whereby their priorities are configured in a complimentary rather than conflicting manner.

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This research examines the details of the relationship between project total cost and duration form the perspectives of both the contractor and the client. The components comprised in each cost-time trade-off curve are articulated and both cost-time trade-off curves are represented mathematically. These curves are then used to develop two further mathematical expressions representing two modes of cost-time compromise between the contractor and the client.

PROJECT COST-TIME TRADE-OFF

The relationship between project cost and project duration has been a matter of concern to researchers since the early 1960s. The fundamental complexity of cost-time trade-off examination stems from the need for extensive and specific data for the analysis. The data need to be extensive in order to reflect the underlying trend representing the overall relationship between cost and time. Also, the data need to be specific to the particular definition and circumstances of the project.

Since each and every construction project is unique, the generation of a large number of hypothetical pairs of cost-time solutions could be somewhat problematic. It is unlikely that a project is repeated many times, each with a different cost-time scenario, covering a range of possibilities from the least to the highest cost-time solutions.

There have been many attempts to overcome this problem. For instance, Fullkerson (1961), Kelly (1961), and Mayer and Schaffer (1965) used the linear programming optimisation technique to formulate the relationship between the two variables. These works laid the foundation for future research by Cusack (1984, 1985a) where he generates the most optimised solution for project cost by focusing on the 'breakthrough' points only. Further elaboration relied on the use of heuristic methods in order to explain the uncertainties associated with the most optimise solution (Cusack 1985b). In parallel with these developments, the work by Phillip and Dessouky (1977) experimented with generating data through activity crashing. Despite alternative simplified methods such as the use of Monte Carlo simulation for generating data (Howes *et al.* 1993), the activity crashing technique has remained as the most viable.

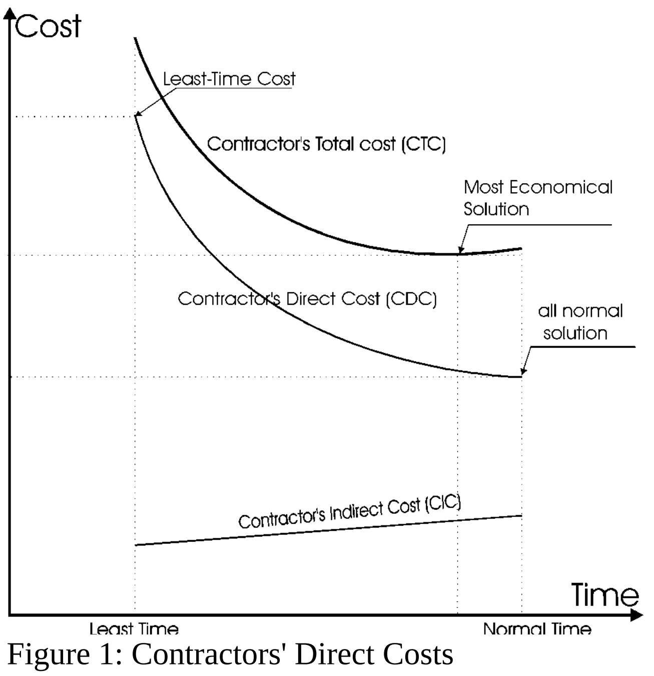

Figure 1: Contractors' Direct Costs

PROJECT COST-TIME TRADE-OFF CURVES

The Contractor
As shown in Figure 1, the contractor's cost-time curve is made up of the superimposition of the contractor's direct costs and indirect costs.

Contractor's Direct Cost [CDC]
This is the main component of the overall contractor's cost-time curve. CDC is generated through the activity crashing exercise.

Activity Crashing
Basically, the activity crashing technique is used to simulate a variety of cost-time scenarios for a given project. As shown in Figure 2, the simulation is accomplished by selecting and crashing (compressing) each critical activity. The selection is based on finding activities with steepest cost-time slopes, as they yield higher changes in cost per unit of compression. At each stage a new pair of project total cost and time is generated. This is continued until the activity reaches the full-crash status (see Antil and Woodhead, 1990). During the activity crashing exercise, it is likely that some previously non-critical activities may become critical, in which case, the crashing will also apply to them.

For the crashing exercise, the relationship between activity cost and duration is assumed to be linear (Kelly, 1961, Fulkston, 1961 and Cusack, 1985b).

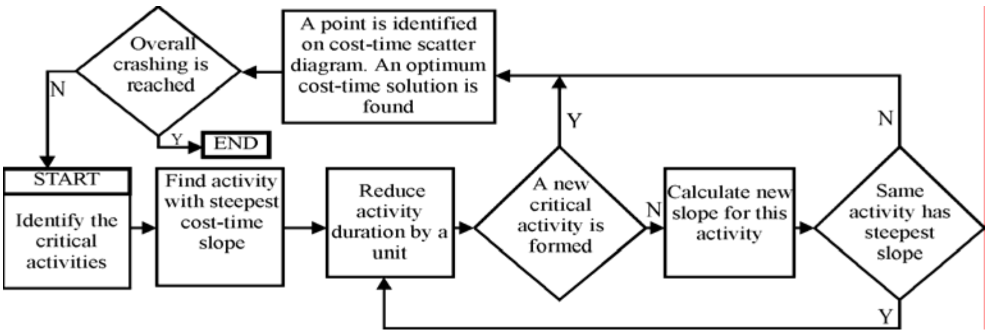

Figure 2: Procedure for Activity Compression

The application of this technique to the activities of one project is likely to generate a number of cost-time alternatives. However, often the generated number of pairs is not enough to create the cost-time curve. Further, the use of one project is unlikely to generate data that is representative of that particular type of project. Therefore, the exercise is applied to a number of similar projects. For each activity of each project, a series of cost-time points are generated. The combination of all points generated from all projects provides adequate number of points for the activity. Subsequently, a linear regression is fitted to the data and the resulting regression model can be used to generate adequate data. This process is repeated for each critical activity and those likely to become critical during the process.

It should be noted that the aggregation of all projects requires that they are standardised into a series of common set of activities. An example is shown in Table 1.

Table 1: Standardised Activities

Substructure *Superstructure* *Carpenter & Joinery*	*Window glazing* *Wet work* *Painting*	*Plumbing & drainage* *Electrical* *Fire service Installation*	*Lift installation* *Prelim*

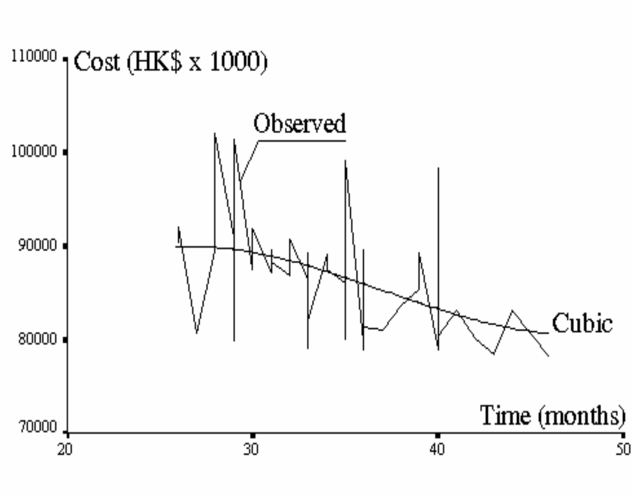

Figure 3: Regression Line: CDC Curve

Once, an adequate number of project cost-time points are generated, a non-liner regression line is fitted to the data. Normally, a quadratic fit will suffice, but sometimes a cubic fit provides a better representation. Figure 3, shows an example of a cubic fit applied to Harmony Housing projects in Hong Kong. On this occasion the regression model produced an R-Squares value of 99.7%. (Khosrowshahi, 1997).

This relationship is generally expressed as follows

(1) $\mathbf{CDC = a_1 + b_1\, T + c_1\, T^2 + d_1\, T^3}$ a_1=constant, b_1, c_1 & d_1 are coefficients

Contractor's indirect costs [CIC]:
These are represented by the preliminary items such as field management, plant, temporary site facilities and head office support (Hawkny 1965). CIC is a function of time and assumes a linear form.;

(2) $CIC = a_2 + b_2\, T$

Hence, contractor’s total costs **CTC** = CDC + CIC.

(3) $CTC = a_1 + a_2 + (b_1 + b_2)\, T + c_1\, T^2 + d_1\, T^3$

The Client
As shown in Figure 4, the client's total costs (**LTC**) is comprised of all the costs incurred by the contractor plus additional costs consisting of the contractor's profit, interest charges, and other time-related costs.

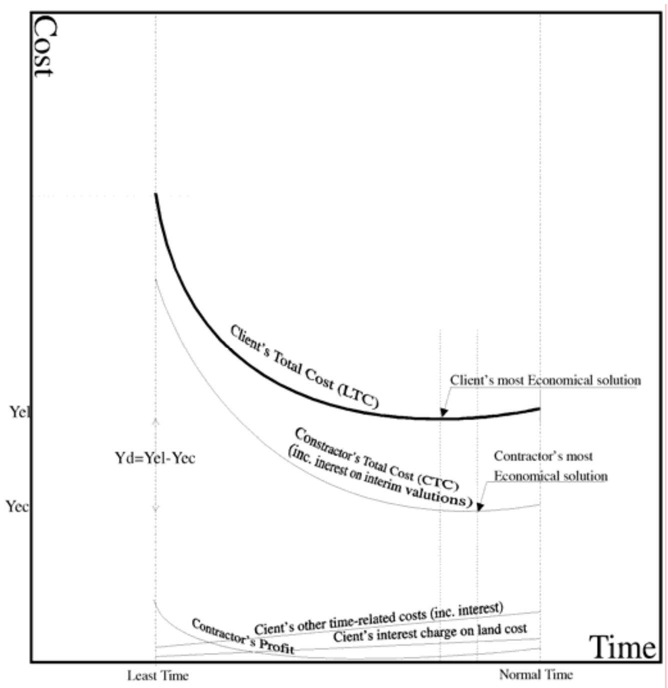

Figure 4: Client's Total costs (LTC)

Contractor's profit **(P);**

Measured as % of contractor's total cost

Interest Charges on Cost of Land [I_l].

The relationship between interest on land costs [I_l] and duration [T] is expressed linearly as;

(4) $\mathbf{I_l = a_3 + b_3 T}$ a_3 and b_3 are constants

To construct this equation, two points need to be identified. These are the interests at normal-time and least-time. Here, the compound interest calculation applies.

$\mathbf{L * (1 + I/100)^T}$ I = average interest rate

L = cost of land (which can be expressed as a proportion of client total cost)

Interest on Capital-in-use [INT_{ciu}]

In order to fund the project, the client requires cash. This is either provided through borrowing or they are supplied from the reserves. Therefore, the company either pays interest or is deprived from earning interest. Normal interest charge calculation is used to evaluate the role of interest charges. Project cash flow often assumes a non-linear form and there are many methods for their forecast (e.g. Khosrowshahi 1993). Here, for simplicity of calculation, linear relationship is assumed. The interest charge calculation is given as follows.

(5) $INT_{ciu} = (CTC)(I+P)(I_c/100)$

I_c= .5 * interest rate * project duration

Other Time Related Costs [V_i]

There are other costs such as professional management and supervisory fees and loss of income due to extended project duration (negative income). These costs are assumed to increase linearly with time and they are calculated as follows;

$\Sigma V = V_1 + V_2 + . . V_i + . . + V_n$ where, $V_i = a_i + b_i T$

These costs too are subject to interest charges at the rate of I_i. Averaged over half the period of application, the interest on Vi is;

$(a_i + b_i T)(0.5 I_i/100)$.

Therefore, the total of clients other time related costs and their respective interest charges are;

(6) $V_{int} = (a_i + b_i T)(1 + .5 I_i/100)$

Hence, the client's total cost **LTC** is the summation of the above costs.

(7) $LTC = (CTC)(1 + P) + INT_{ciu} + I_l + V_{int}$ *P=profit/100*

$= (P+1)(1+.01I_c)(a_1+a_2 + (b_1+b_2)T + c_1T^2 + d_1T^3) + (a_3+b_3T) + (1+.005I_i)(a_i+b_iT)$

$= x[(a_1+a_2+a_3/x + ya_i/x) + (b_1+b_2+b_3/x+yb_i/x)T + c_1T^2 + d_1T^3]$

where $x = (P+1)(1+.01I_c)$ and $y = (1+.005I_l)$

THE MOST ECONOMICAL SOLUTION

The most economical solution for the contractor [Y_{ec} & T_{ec}] and for the client [Y_{el} & T_{ec}] are calculated by identifying the project duration that corresponds to the point where the first derivative of the LTC is equal to zero.

The Contractor

From equation 3, the relationship between contractor's most economic cost [Y_{ec}] and its corresponding duration [Y_{ec}] are given below;

8) $Y_{ec} = a_1+a_2 + (b_1+b_2) T_{ec} + c_1 T_{ec}^2 + d_1 T_{ec}^3$

$\partial CTC / \partial T = 0 = b_1+b_2 + 2c_1 T + 3d_1 T^2$

Hence

$$T_{ec} = \frac{-2c_1 \pm \sqrt{4c_1^2 - 12d_1(b_1+b_2)}}{6d_1}$$

There are two solutions for T_{ec}. The solution, which produces a positive result, when applied to the second derivative of equation 3, ($d^2CTC / dT^2 = 2C_1 + 6d_1T$) gives the most economical solution. Then T_{ec} can be inserted into equation 8 to give the most economical cost;

The Client.

From equation 7, the relationship between client's most economical cost [Y_{el}] and its corresponding duration [T_{el}] are given below;

(9) $Y_{el}=x[(a_1+a_2+a_3/x+ ya_i/x) + (b_1+b_2+b_3/x+yb_i/x)T_{el} + c_1T_{el}^2 + d_1T_{el}^3]$

$\partial LTC / \partial T = 0 = b_1+b_2+b_3/x+ yb_i/x + 2c_1 T + 3d_1 T^2$

Hence

$$T_{el} = \frac{-2c_1 \pm \sqrt{4c_1^2 - 12d_1(b_1+b_2+b_3/x+yb_i/x)}}{6d_1}$$

The valid solution is identified by inserting T_{el} values into the second derivative of equation 7 ($d^2LTC / dT^2 = 2C_1 + 6d_1T$). Again, we are interested in the T_{el} that produces a positive result. The value for T_{el} is then inserted into (9) to calculate Y_{el}.

CONTRACTOR-CLIENT COST-TIME COMPROMISE SOLUTIONS

As noted earlier, due to their varied priorities, there may be circumstances where the client and the contractor agree to negotiate a compromise solution. Theoretically

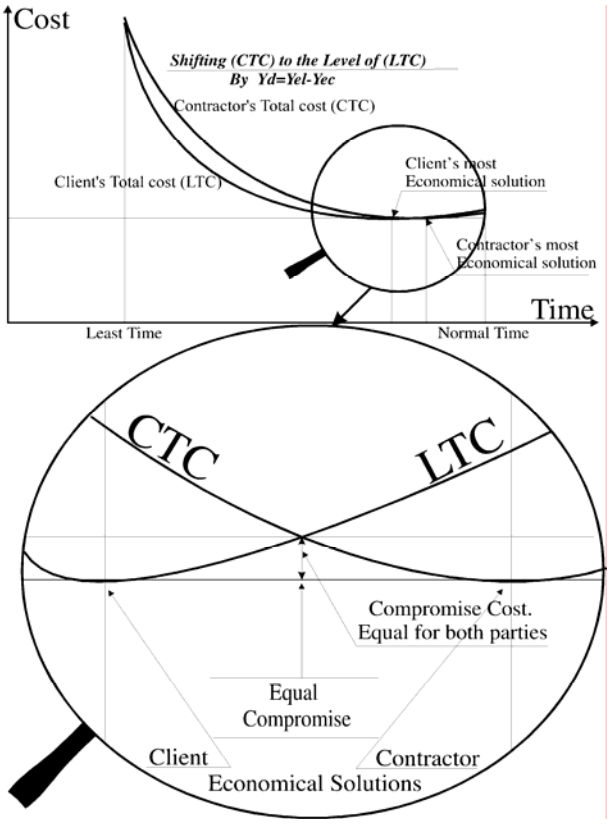

Figure 5: Equal Compromise Solution

speaking, there are infinite cost-time scenarios that exist between the client's most optimum solution and the contractor's most optimum solution. Obviously, in any negotiating situation, the parties can arbitrarily select and agree on a cost-time solution. However, as well as the two extreme points (the client and contractor optimum solutions) there are two other cost-time solutions that can be optimised. These two solutions involve both the client and the contractor, thus, their consideration may be of mutual interest to the parties. These options are discussed below and their corresponding generalised mathematical expressions are developed.

Equal Compromise Solution

In this compromise option the parties agree to share the compromise on an equal basis. In other words, the project duration would be such that both parties deviate from their respective minimised costs by an equal amount. In order identify the equal compromise solution, the client's total cost curve (LTC) is physically lowered by Y_d amount, to a new position (LTC_{shift}) whereby both optimum solutions are placed on the same horizontal line. This is shown in Figure 5.

Since $Y_d = Y_{el} - Y_{ec}$

$LTC_{shift} = LTC - Y_d$

For duration $T_{e\text{-}compromise}$ (or $T_{e\text{-}c}$), the equal-compromise cost will be produced. This relates to the point where CTC and LTC_{shift} intercept.

Therefore, the equal-compromise cost is calculated by equating CTC and LTC_{shift} for $T=T_{e\text{-}c}$.

From 7 & 10 $LTC_{shift} = LTC\text{-}Y_d = CTC(1+P) + INT_{ciu} + I_l + V_{int} - Y_d$

Set $CTC(1+P) + CTC(1+P)(.1I_c) + I_l + V_{int} - Y_d - CTC= 0$

Hence $CTC\ [P + (1+P)\ (.01I_c)\ ] + I_l + V_{int} - Y_d = 0$

Hence

$[P+(1+P)(1+.01I_c)]\ [a_1+a_2 + (b_1+b_2)T_{e\text{-}c} + c_1T_{e\text{-}c}^2 + d_1T_{e\text{-}c}^3] + a_3+b_3T_{e\text{-}c} + (1+.005I_i)(a_i+b_iT_{e\text{-}c}) - Y_d=0$

Hence $a_1+a_2 + (a_3+ya_i\text{-}Y_d)/x + (b_1+b_2+(b_3+yb_i)/x)T_{e\text{-}c} + c_1T_{e\text{-}c}^2 + d_1T_{e\text{-}c}^3 = 0$

where $x= [P+(1+P)(.01I_c)]$ & $y= (1+.005I_i)$

There can be three values for $T_{e\text{-}c.}$ The valid solution is the positive value, which yields the lowest cost.

Optimum Compromise Solution

Any deviation from the optimum solution of the contractor or the client, increases the project cost for that party. Similarly, any deviation from the equal compromise duration yields financial advantages for one party at the detriment of the other party. However, there is a particular project duration, which minimises the total sum of compromises by both parties. In comparison to the equal compromise solution, the optimum compromise solution is likely to favour one party over the other.

As shown in Figure 6, the duration ($T_{o\text{-}compromise}$ or $T_{o\text{-}c}$), which minimises the total sum of the compromises for the client (H_l) and the contractor (H_c), relates to the optimum-compromise solution. In order to minimise the aggregate of H_c and H_l, the derivative of the equation of (CTC + LTC = S), must be set to zero.

$S = CTC+LTC = 2CTC +(CTC)P + INT_{ciu} + I_l + V_{int}$

$= [(2+P)+(1+P)(.01I_c)][a_1+a_2 +(b_1+b_2)T +c_1T^2 +d_1T^3] + a_3+b_3T + (1+.005I_i)(a_i+b_i)T$

$= x[a_1+a_2 +(a_3+ya_i)/x +(b_1+b_2+(b_3+yb_i)/x)T + c_1T^2 + d_1T^3]$

where $x= [(2+P)+(1+P)(.01I_c)]$ & $y= (1+.005I_i)$

Set $\partial S / \partial T = 0 = b_1+b_2+(b_3+yb_i)/x + 2c_1 T + 3d_1 T^2$

Hence, solve for time (or $T_{o\text{-}c}$)

$$T_{o-c} = \frac{-2c_1 \pm \sqrt{4c_1^2 - 12d_1(b_1+b_2+b_3/x+yb_i/x)}}{6d_1}$$

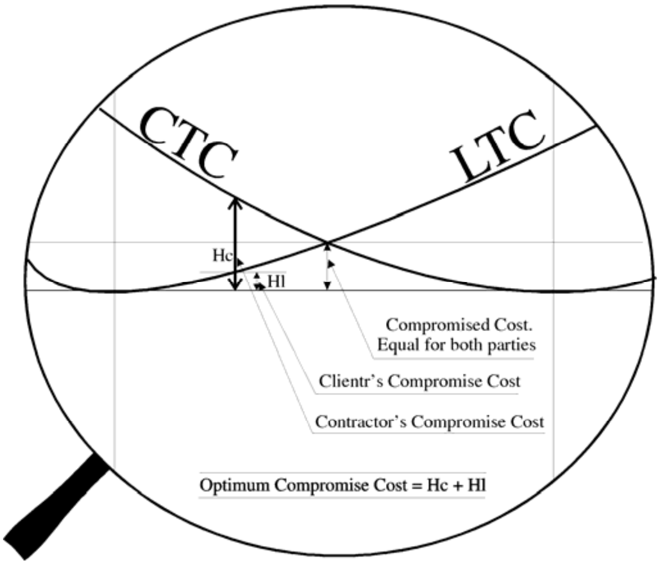

Figure 6. Optimum Compromise Solution

There are two possible durations that will produce optimum-compromise solutions. Alternate choice of these durations will switch the favour from the client to the contractor and visa versa.

CONCLUSIONS

The contractor's Cost-time trade-off curve are examined in greater detail and contrasted against the client's cost-time trade-off curve. For both curves, the generalised mathematical expressions are developed, taking into account all possible costs contributing to both the client and contractor's cost-time curves. Situations were discussed where the parties might negotiate and agree on a compromise cost-time solution. To this end, an infinite number of possibilities exist in the range between the client's most optimum duration and the contractor's most optimum duration. The choice of any solution within this range is likely to increase the project cost for one party at the cost of the other. However, if the parties agree to negotiate a compromise solution then there are two cost-time solutions that can be optimised: *equal compromise solution* and o*ptimum compromise solution*. While the former option enables the parties to equally divide the compromises, the *optimum compromise solution* minimises the total sum of compromises sustained by both parties. The generalised mathematical expressions for these solutions are also developed.

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