PRESERVING CONTINUITY IN WHOLE-LIFE COST MODELS FOR NET-ZERO CARBON BUILDINGS

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Investment in net-zero carbon buildings requires comprehensive evaluation especially with regards to economic viability. Mathematical modelling of whole-life costing provides a relevant framework to assess the investment potential of net-zero carbon buildings. Previous studies in investment analysis have suggested insufficiency in the discounting mechanism of cash flows leading to unrealistic estimation, and in some instances, incorrect decisions. There is a growing body of evidence that conceptual adjustments to cost models could facilitate improvements in the costing of zero carbon buildings. This study, - which is part of a PhD investigation on cost studies in zero carbon buildings, presents an approach to preserving continuity in whole-life cost models using the binomial theorem. The work builds on the New Generation whole-life costing developed in Ellingham and Fawcett (2006) by extending the period under consideration and concurrently providing for other elements of time, uncertainty and irrevocability. The study also highlights the conceptual importance of continuity in decision-models. An illustrative costing exercise is carried out, over a 25-year period, on a conventional and net-zero carbon building using three different whole-life cost procedures. Results from the study suggest that continuous whole-life cost models provide a realistic template for representing cost variables especially in comparative studies. Future research will examine the implications of continuous whole-life costing for a generic net-zero carbon building. This will provide construction professionals with clear aspirational objectives on the economic performance of net-zero carbon buildings.

Keywords: continuity, cost models, present-value, whole-life costing, zero-carbon

INTRODUCTION

In December 2006, the UK Government announced zero-carbon compliance for new housing and schools by 2016; public sector buildings from 2018 and commercial buildings from 2019. It is expected that the zero-carbon agenda would induce changes in the supply chain of the construction industry. Some of the anticipated changes in the supply network might include the “change in role(s)” for existing parts, displacement of vestigial units as well as admittance of new members. These changes should present an opportunity for the construction industry to re-strategize and re-position itself towards becoming a more efficient sector in the delivery of modern sustainable buildings. It is not unreasonable to expect the costing of housing units to play a pivotal role in this drive. Moreover, there are marked indications that benefits

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from an improved costing framework are likely to be far-reaching in this developmental epoch (Smit 2012). A number of notable industry reports have previously mentioned poor performance of the housing and construction sector resulting in unprofitability, unpredictability and poor client satisfaction (Egan 1998; Barker 2004). It is anticipated that these trends will be improved upon as momentum gathers in the advance towards a net-zero carbon built environment.

Investment in net-zero carbon buildings requires comprehensive evaluation in order to ascertain economic viability. Many published studies have already hinted at higher capital cost implications for net-zero carbon buildings compared to conventional buildings (Clarkson & Deyes 2002; Catto 2008; Williams 2012). CLG (2008) reported that an additional construction cost of 25-37% was needed to achieve “level 6” of the Code for Sustainable Homes (also known as zero-carbon standard), in new buildings. This figure closely tallied with the findings of Holden and Twinn (2011) that net-zero carbon houses could be about 40% more expensive in the build-cost requirement. A later study by Williams (2012) contends that the additional capital investment cost in net-zero carbon houses might not exceed 10%, given the competence and aptness in the scale of operation of the house developer. Considering the propositions in these disparate accounts and especially in the current economic climate, analytical costing of zero-carbon buildings could play a germane role in identifying the relevant cost drivers. Analytical approach to costing could also aid an appreciation and understanding of the entire process relating to delivery of net-zero carbon buildings. This paper presents an innovative and analytical approach to costing net-zero carbon homes. Critical reflections on the mathematical modelling of costing as well as the binomial theorem, constitute the background literature for the investigation. An overview of the distinct variants of whole-life cost techniques are stated and appraised. A comparative illustration employing the procedures of the whole-life cost techniques is also presented. The results obtained are collated and reported upon in the final section of the paper. Implications of the results from the cost models are discussed and employed in suggesting future research directions.

**MATHEMATICAL COST MODELS**

A mathematical cost model is a unit of analysis which consists of sets of relationships, systematically arranged to handle inputs and methodologically translate them into outputs (Smit 2012). According to Farr (2011), represented in Figure 1 below, the paths to mathematical modelling are essentially a choice between closed-form or finite elements methods. Ross (2009) suggested that closed-form mathematical expressions provide precise descriptions for systems with little complexity and hence assume little uncertainty. Current trends in cost estimation however suggest increasing complexity, heightened uncertainties and relative noisiness of data (Boussabaine & Kirkham 2008). In cost estimation, an alternative to building mathematical models is the use of simulation which could be in the form of system-dynamics or Monte Carlo (Farr 2011). Simulations can provide a cheaper way to conduct a simplified analysis of a system over a specified period of time (Farr 2011). Simulations are highly beneficial in simplifying the characteristics of a system and obtaining reasonable expectancies on system performance. One demerit of simulation is that it is not an optimizer, but provides satisfactory solutions (Wayne 1996; Boussabaine & Kirkham 2008). It is also quite evident that simulation experiments seldom tend to establish fundamental relationships; rather they juggle an array of input variables in order to determine the impact of their possible combinations on one or more output variables. Wayne (1996)
equally acknowledged the advantages of mathematical models in place of simulation experiments but contends that for many a system, a true mathematical model does not exist. Arguably, this claim does not seem to have been empirically substantiated.

![Figure 1: Paths to Mathematical Modelling. (Source: adapted from Farr 2011)](image)

Existing trends however point at an increasing and persistent gap between the predictions of cost models and eventual economic realities (Boussabaine & Kirkham 2008). Ellingham and Fawcett (2006) reasoned that many managers understand that numerous cost models are flawed and often make informal adjustments to compensate for the deficiencies in cost estimates. The demerit of such an approach is the arbitrariness in the magnitude and impact of adjustments made. Ellingham and Fawcett (2006) further expressed that flawed cost models resulted from a consistent lack of recognition of life-cycle options embedded or acquired in decision choices. Clarkson and Deyes (2002) had also earlier noted that cost studies were often conducted in a limited intertemporal optimization framework. Georgiadou and Hacking (2011) pointed out that many of the existing cost figures are based on deterministic projections of historical data which can only provide “steady-state” models; and thus have little bearing on reality. These accounts sufficiently suggest a growing body of evidence seeking to establish the analytical underpinnings of contemporary cost models. Mathematical modelling provides a robust means of implementing and facilitating analytical significances in costing (Ellingham & Fawcett 2006; Farr 2011; Smit 2012).

**CONTEMPORARY ISSUES IN COST MODELLING**

For costing to be carried out it is important to understand the objectives and requirements of the particular system, as well as its constraints and assumptions (Smit 2012). In cost modelling, like every other physical and engineering system analysis; evaluation needs to be conducted across a robust frame of reference. In many cost modelling exercise, time is perhaps the most prevalent frame of reference, especially in life-cycle scenarios. Ayyub (1999) added that the development of a cost model also results in introducing and defining uncertainties. Uncertainties, on the one hand consist of lack of information which could emerge from cognitive or non-cognitive sources (Ayyub *ibid*). Core areas of uncertainty in estimating cost across a product’s lifecycle include cash flow data, building-life, investor’s commitment, component service life and future decisions (Ellingham & Fawcett 2006). On the other hand, recent research has also presented a case for the existence of a significant degree of economic and/or physical irrevocability in projects; evidence of this, is seen in some literature on housing (Verbruggen *et al.* 2011; Smit *ibid*). In the context of buildings, irrevocability can be termed a “lock-in” syndrome (CLG 2011). This implies that once built, a certain level of efficiency or inefficiency is locked into a building which cannot be dramatically altered without significant and disruptive costs. Irrevocability therefore connotes the difficulty and/or impossibility associated with withdrawing resources already committed to a course of action for an alternative use. Verbruggen *et al.* (2011) represented a four-degree irrevocable process in buildings; very strong, strong, medium and weak. Very strong connote situations where the cost of reversal increases over time. Strong refers to those where reversal cost in the future is above the reference initial cost but decays over time; medium refers to reversal cost being
higher than initial cost at current time and for some years but falling below initial cost in later periods; Weak refers to reversal cost being equal to or less than the initial cost.

Supposing time, uncertainty and irrevocability are attributes worthy of being represented in cost models, the analytical underpinnings of such procedures are not very straight-forward. The discounting process is the widely-accepted mechanism of deriving the equivalent value today of a future expenditure (Park & Sharp-Bette 1990). Previous studies in investment analysis have however suggested insufficiency in the discounting mechanism of cash flows leading to unrealistic estimation and in some instances, incorrect decisions (Gluch & Baumann 2004). Korpi and Ala-Risku (2008) have also questioned the discounting convention which invariably elevates the place of running cost over initial capital cost. Chan (2012) hinted that the problem with the conventional discounting mechanism might be embedded in the cultural perception of time as a homogeneous numerical order. Kishk and Al-Hajj (1999) have reportedly cautioned that costing does not completely fit into the framework of probability and statistical theories. Hence, there is need to expand the purview of modelling in order to augment the needed robustness and flexibility in cost evaluation.

Perceptibly, ZCH (2011) has expressed that cost modelling could assist in benchmarking the occurrences in a housing project which could then serve as a proxy for feasibility assessments. One effort which also appears promising especially in the containment of irrevocability is recognising the continuity attribute in cost models. The principle of continuity is a methodical approach underlying the conceptual notion that reality is a dynamic sequence of events and decisions (Verbruggen et al. 2011). By and large, the principle of continuity models progressive and successive events as being intrinsically interlinked. Continuity, an age-long mathematical principle, can be considered as an aspect in finite element (FE) algebraic analysis of cost models.

**BINOMIAL COST MODELLING**

Analytical costing presupposes the existence of a representative cost function for a costing process. Whilst there may be differing opinions on the existence of a true cost function, such assumption is fundamental to employing mathematical models in costing. In many costing situations, data is known only at discrete points. However, the value of a function at a non-discrete point may be required to better understand the behaviour of a cost system (Hoffman 2001). In such situations, one pragmatic endeavour becomes fitting an approximate function to the set of discrete data. For the purposes of simplicity and ease of manipulation, polynomials are often an excellent choice in fitting an approximate function (Hoffman *ibid*). Furthermore, the binomial model is the simplest form of polynomial in any given probabilistic sample space. The binomial model shown in Fig 2b below is a mathematical representation of the rate of change of cost with respect to time \(\frac{\delta C}{\delta t}\) and explicitly recognizes a stream of possible values within a sample space. The binomial model also specifies probability coefficients for respective cost values based on the differentiation calculus. In obtaining the empirically derived cost projections, this work builds on the binomial model developed by Ellingham and Fawcett (*ibid*). In the binomial model, the normalized coefficients of each term follow the sequence as shown in Figure 2a:
The equation below is a general binomial series and can be represented as:

\[ nC_0 + nC_1 + nC_2 + \ldots + nC_n \]  

(Eqn. 1)

Also for any term, \( r \) and column, \( n \), the general equation can therefore be expressed as:

\[ nC_r = \frac{n!}{(n-r)!r!} = \frac{nC_{r-1}}{nC_0 + nC_1 + nC_2 + \ldots + nC_n} \]  

(Eqn.2)

According to Ellingham and Fawcett (\textit{ibid}), the binomial theorem can be used to forecast, rather than predict future uncertainties. In the binomial model in Figure 2a (Pascal’s triangle), the \( r \)th term, \( n \)th column as well as the number of binomial coefficients, \( k \) on the sample space, is used to deduce the normalized coefficient of the binomial model. The mathematical formulation employed in deriving the individual probability equivalent for each cost index is deducible by the formula given as:

\[ \frac{n-1C_{r-1}}{\sum_{k=0}^{n-1} n-1C_k} \]

One obvious benefit in mathematical-based binomial cost models is the consistent and explicit calculus used for evaluating risks over a specified period of time. This permits the effects of inflation and discounting to be separately and comprehensively dealt with; and also facilitates robust and procedural evaluation of each mechanism.

**STANDARD WHOLE-LIFE COSTING (WLC)**

Over the last few decades, there has been a significant appeal for costing to be extended across the entire life time of projects (Gluch and Baumann 2004; Kishk 2005; Smit 2012). Mathematical modelling of whole-life costing provides a relevant framework for assessing the investment potentials in constructed built facilities. According to the CIFPA (2011), whole-life costing is simply the systematic consideration of all relevant costs and revenues associated with acquisition and ownership. The Net Present-Value (NPV) is the common metric for assessing the whole-life cost of construction projects, and is sometimes regarded as the whole-life cost of a building (Kishk 2005). In mathematical terms, WLC can be represented as:
Conceptually, the whole-life costing mechanism compares a range of existing options, leading to a “choose” or “lose” situation, in which, one of the evaluated options often translate into overall better investment based on the estimation of future revenues and costs. The predominant elements in whole-life costing procedures are initial capital costs, running costs, interest rates and inflation rates. Other specialized models further separate cost elements into procurement, maintenance, repair and operational costs (Kishk *ibid*). Usually - for purposes of ease and convenience, many whole-life costing exercises separates costs into just “initial capital” and “running” cost categories. Kishk and Al-Hajj (1999) expressed that by separating costs into capital and running categories, a peculiarity has been established. Some researchers have however criticised such simplistic approach. Tietz (1987) illustrated a situation in which the running cost of a building estimated over a 50-year period is likely to be 0.8 – 1.3 times the capital cost, assuming a discount rate of 2% above inflation. Assaf *et al.* (2002) also contends that the relationship between “capital cost” and “running cost” is essentially unknown. Ferry *et al.* (1999) stated that it is inappropriate to attempt to equate initial and running cost, since the circumstances and benefactors of both costing elements are often different. Other variants of the standard whole-life cost formula have been well documented by Kishk (*ibid*). Kishk (*ibid*) conjectured they were based on the same closed-form expression. Park and Sharp-Bette (1990) earlier inferred that closed-form expressions typically converge to a particular value.

There have however been a number of concerns on the appropriateness of the standard whole-life cost framework in providing accurate long term forecast of all associated costs since it is based on the discounting mechanism (Kirkpatrick 2000; Kishk *ibid*). Perhaps in recognition of such concerns on the performance of WLC models, a UK Government report issued by the Building Research Establishment and reported in Clift and Bourke (1999) identified several barriers to applying whole-life costing, namely: its lack of universal methods and standard format of computations; the absence of a stipulated methodology for integration of operation and maintenance strategies at the design phase, as well as the large scale of the data collection exercise. The CIFPA (*ibid*) however expressed that the ultimate value of whole-life costing lies in improving the understanding of the key links and drivers between the initial purchase decision and future costs and benefits.

**NEW GENERATION WHOLE-LIFE COSTING (NWLC)**

The New Generation whole-life costing introduced by Ellingham and Fawcett (2006) is an experimental departure from the standard whole-life costing procedures. One crucial motivation behind this new-generation whole-life cost methodology is the incongruence in the outcome of whole-life costing analysis and the gut-feeling of decision-makers. Ellingham and Fawcett (*ibid*) argued that by relaxing the rigid assumptions of standard whole-life costing - that all decisions are made in year 0, and are irrevocable - increases the whole-life value. According to Verbruggen *et al.* (2011) this brand of costing is an application of the “wait and learn” scenario of the real option literature. “Options-thinking” is basically a conceptual idea that certain decisions can be taken in the future with better information. The life-cycle option described here is analogous to financial options and derives directly from the Black-
Scholes equation for establishing the fair price of an option (Ellingham & Fawcett *ibid*). Life-cycle options are basically the opportunities to respond to future change.

The present-value (PV) of the entries in the binomial cost model is the discounted weighted average of all the entries. The procedures for deducing the present values for each respective year have been described in Ellingham & Fawcett (2006). As an illustration, the formula employed to evaluate the binomial tree for year 1 in Fig 3b is:

\[
PV = v + P_u \times \frac{V_u}{(1 + d)} + P_d \times \frac{V_d}{(1 + d)} 
\]

*(Eqn. 5)*

Where,
- \(PV\) = Cumulative value at particular node ; \(v\) = Income at the particular node,
- \(V_u\) = Cumulative value at the upward parent node; \(P_u\) = Upward probability
- \(V_d\) = Cumulative value at downward parent node; \(P_d\) = Downward probability
- \(d\) = discount rate

The present-value tree, NPV of the development tree and option tree are then computed. To calculate the NPV of development and option value for the model, a roll-back mechanism similar to a decision-tree analysis is carried out. The New Generation whole-life cost is perhaps most relevant in situations where there is a choice to expand, refurbish, sell, switch use, or include new technologies over a long period of time (Ellingham & Fawcett *ibid*). The New Generation whole-life cost approach is beneficial in its intellectual stimulation and assists clients to explicitly visualise the rationale behind costing decisions. It also encourages systematic consideration of cost information. Also, the procedures could be implemented on a spreadsheet package.

Some of the limitations with the new generation whole-life costing model are that its procedures are more rigorous than standard whole-life cost mechanisms and might require expert guidance to be properly implemented. Also, the data across the estimated years are adjusted through the roll-back mechanism, but the model itself is neither dynamic nor adjustable in the decision framework.

**CONTINUOUS WHOLE-LIFE COSTING (CWLC)**

There has been a growing proclivity towards continuity in evaluation models in the literature on engineering and construction economics (Park & Sharp-Bette 1990; Chan 2012). The extent and methodology however remains a technical difficulty to be subdued. An earlier work exploring continuity in cash flows was reported by Park &
Sharp-Bette (*ibid*). Park and Sharp-Bette (*ibid*) reckoned that continuity could occur in two main areas namely: continuous compounding of cash flows and representation of cash flows as proceeding at a given rate continuously, as opposed to discrete periods. The procedures reported by Park and Sharp-Bette (*ibid*) however tacitly stereotypes and adopts the exponential growth profile typical with the conventional discounting function and as such provides a somewhat analogous logic to the WLC mechanism. In following an exponential pattern without providing for other possible alterations, cost values are not open to being adjusted based on emergent market realities. The binomial theorem calculus, though not without its limitations, provides a platform for adjusting the continuous compounding of cash flow projections.

The CWLC framework introduced here aims to build on the template of the new-generation whole-life costing through preserving continuity in the NWLC equation. In contrast to Verbruggen *et al.* (2011) the authors reckon that the decision criterion of a (net-zero carbon) building are not always a stiff choice between a “choose or lose” and the “wait and learn” scenario. Essentially, a feasible continuum exists between the expenditure choices at the disposal of clients and the timing of making such decisions. It is possible to propose the existence of a progressive rate of change of cost with time (Park & Sharp-Bette *ibid*). In many established economies, the rate of change of cost is assumed linearly progressive with time. The stream of potential revenues can be derived using the binomial-based cost model template. In this CWLC model, components of the initial capital and running cost categories are retained. The linearity assumption aids analytical ease and facilitates compatibility with the binomial theorem framework. In modifying the binomial cost structure developed by Ellingham and Fawcett (2006), continuity is infused over the estimated life; in this case, 25 years. The CWLC equation in the proposed approach is presented as:

\[
\text{CWLC Equation} = C_0 + \int_{t=0}^{n=25} C_k(t) \, dt
\]  

*(Eqn. 6)*

By expanding the integral we obtain the equation:

\[
C_0 + \left[ tC_k \right]_{t=0}^{n=25} + \sum_{t=0}^{n=25} \delta c \delta t = C_0 + \left[ tC_k \right]_{t=0}^{n=25} + \delta t \times \sum_{t=0}^{n=25} \delta c
\]  

*(Eqn. 7)*

In traditional integral expansion, the term \( \sum_{t=0}^{n=25} \delta t \delta c \), in eqn 7 is a constant \( K \).

This constant, \( K \), is often considered infinitesimal in most mathematical integration procedures and often approximated to zero or simply considered non-existent. It should however be noted that such approximations, where applicable, are admissible in say, distance-to-time measurements, where the dimensional quantities are in metres and seconds respectively. However in cost models, the dimensional equivalent of time is in years and sums of money are in local currencies; which often have significant digits. Such approximation might be rather inimical to robust model development. The CWLC proposed here intends to dynamically evaluate the cost mechanism through a procedure that obtains the exact difference in the average binomial value computations for each successive year \( \delta c_a \) as the project progresses through its life cycle. The first step in this procedure involves the estimation of a stream of binomially generated revenues across the estimated number of years \( n \) and averaging its value to yield each incremental cost value. The incremental cost is progressively summed over the specified number of years to attain the cost difference, \( \delta c \), which is
This summation is cumulatively added to the initial capital cost and the average running cost for the estimated period. The continuous whole-life cost figure is basically the cumulative summation of the incremental running costs, the initial capital cost and the average running cost for the estimated number of years.

**NUMERICAL ILLUSTRATION AND DISCUSSION**

For the purposes of practical illustration, this study evaluated the initial capital and running cost data of a net-zero carbon house (*Lighthouse*) and conventional house over a period of 25 years using variants of three different whole-life cost equations. According to Cook (2011), the build cost of the *Lighthouse* was £75,000, excluding the cost of foundations and utility service connection. Catto (2008) earlier hinted that possibility abounds for a net-zero carbon house to be constructed at a cost of £120,000. Holden and Twinn (2011) have also found that the running cost of the first zero carbon house (*Lighthouse*) in the UK is £30 per year, in contrast to the £500 per year that will be incurred with a conventional house, of similar capacity; which complies with the previous part L, Building Regulation, 2006. Catto (*ibid*) expressed that such conventional building could cost about £85,000, which roughly represents an approximate 40% reduction from the approximate cost of a zero carbon house. Juxtaposing these accounts, this study adopts a capital cost of £120,000 and running cost of £30 per year for a zero-carbon house and a corresponding capital cost of £85,000 and running cost of £500 per year for a conventional house. Table 1 compares the present-value cost figures of the house-types using standard whole-life cost, new generation whole-life cost and continuous whole-life cost techniques respectively. The inflation rate and interest rate of 2.5% and 8.0% respectively is employed in all cases, in line with the work of Ellingham & Fawcett (2006). In effect, a risk-adjusted discount rate of 5.5% was applied in the standard whole-life costing computation.

**Table 1: Comparative Present-value figures for different Whole-life Cost Techniques**

<table>
<thead>
<tr>
<th></th>
<th>WLC</th>
<th>NWLC</th>
<th>CWLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Carbon House</td>
<td>£120,402.44</td>
<td>£120,321.18</td>
<td>£120,758.79</td>
</tr>
<tr>
<td>Conventional House</td>
<td>£91,707.06</td>
<td>£90,352.43</td>
<td>£97,646.54</td>
</tr>
</tbody>
</table>

The CWLC technique introduced here could be beneficial because it does not require a prohibitive amount of data. Unlike in WLC models, the continuous whole life cost model is sensitive to the estimated life time of the building. The CWLC model is also helpful in identifying the exact point where the combined effect of inflation and interest rate has an optimal impact. This awareness could assist in providing a more realistic and dynamic pay-back period in the evaluation of emerging technologies. In addition, assumptions could be varied for successive years which allows for dynamic visualization of cost drivers. Also, cost factors over the lifetime of a building could be based on more recent market realities without recourse to back-casting. Lastly, the continuous whole life costing described here provides a conceptually simple and mathematically tractable approach to rational investment evaluation.

**CONCLUSIONS**

This study promotes consideration of the issues and approaches available for costing net-zero carbon buildings. The continuous whole-life cost is a novel and innovative approach in financial appraisal and costing. Preserving continuity partly explains the gut-feelings of housing clients that have been hesitant to embrace zero carbon homes based on the leanings from some existing cost models. Given current indications, the
UK Government might need to consider a capital subsidy exceeding £23,112.25 (present-value cost difference between CWLC estimates of the net-zero carbon and conventional house) or its equivalent, in order to promote widespread patronage for a house like the Lighthouse, based on a 25-year product life. The initiative of the UK Government in granting a stamp duty exemption of £15,000, for zero carbon housing which costs below £500,000 proves inadequate in this respect. It can also be observed that the values in the continuous whole-life cost technique are considerably higher than the other two whole-life cost techniques. This might suggest that previous cost over-runs could be partly due to a persistent case of methodological oversights leading to underestimation in housing projects. The results also suggest that continuous whole-life cost models provide a comparable and realistic template for representing cost variables. The limitations in the study are that, only one property type has been considered for the illustration; cost figures are also indicative rather than definitive and have been assumed to be longitudinal data. This work however proposes an analytical basis to costing, which equips construction professionals with clear aspirational objectives on the economic performance of net-zero carbon buildings.

REFERENCES


