# HANDLING THE CONSERVATIVE NATURE OF FUZZY WHOLE-LIFE COST SIMULATIONS

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In a previous paper (Kishk, 2004), a practical procedure has been developed to handle statistically significant data and expert assessments within the same whole-life costing (WLC) model calculation. However, results obtained from this algorithm are typically conservative. The objective of the research work that underpins this paper is to further investigate this conservative nature. First, various causes of this conservative nature are explored and discussed to identify how they can be eliminated. Then, an effective WLC model is developed and implemented into a practical algorithm to tackle some aspects of this conservative nature. Typical results showed its advantage over standard WLC models when dealing with normalized data. First, the model saves the time of preparing the data in the standard format. Secondly, and more importantly, the confidence measures in ranking can be better because of the elimination of the additional uncertainty in the predicted WLCs that may be caused by expanding normalised data to the standard format.

Keywords: financial management, fuzzy set theory, simulation, whole-life costing.

## INTRODUCTION

When whole-life costing is as a decision making tool, an explicit mathematical model is usually employed to calculate the net present value (NPV) of all future costs and benefits of each competing alternative. Then, the choice criterion is that the ideal alternative has the minimum NPV. Because the technique, by definition, deals with the future and the future is unknown, it is crucial to identify effective methods that can recover the information that is present in this uncertain data and assess the faith that can be placed on this information and consequently good and reliable decisions can be made. In doing so, either a probabilistic risk assessment technique, usually the Monte Carlo simulation (MCS), or technique based on the fuzzy set theory (FST) is employed.

MCS has been used in handling uncertainty in WLC data by many authors, e.g. Flanagan *et al.* (1989), Smith (1994), and Goumas *et al.* (1999). In an MCS exercise, every uncertain variable is represented by a probability distribution function (PDF). The resulting whole-life costs become random variables represented by PDFs. As noted by Flanagan *et al.* (1989), this provides the decision-maker with a wider view in the final choice between alternatives but will not remove the need for the decision-maker to apply judgment and there will be, inevitably, a degree of subjectivity in this judgment. Moreover, simulation techniques have been criticized for their complexity and their expense in terms of computation time and expertise required to extract the knowledge (Byrne, 1997; Edward and Bowen, 1998).

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Because mathematical concepts and operations within the framework of FST are much simpler and more effective than those within the probability theory, it has been employed by several researchers to handle uncertainty in WLC studies. In this approach, uncertain variables are represented by fuzzy numbers (FNs) and the resulting whole-life costs become FNs as well. Sobanjo (1999) employed an approximate method proposed by Kaufmann and Gupta (1988) to manipulate fuzzy numbers in WLC calculations. However, Sobanjo's model can deal only with uncertainty in cost variables. Besides, only triangular fuzzy numbers can be used. Moreover, it cannot handle random uncertainties in statistically significant data. The algorithm developed by Kishk and Al-Hajj (2000) tackled the first two shortcomings.

Recently (Kishk, 2004), another algorithm has been developed to deal with various facets of uncertainty in statistically significant data and expert assessments within the same whole-life costing (WLC) model calculation. This allowed comparison between fuzzy and probabilistic approaches to handle uncertainties in WLC modeling. Typical results showed that fuzzy algorithms are more conservative.

The rest of the paper is organised as follows. In the next section, various sources of the conservative nature of fuzzy arithmetic are discussed. Then, various published WLC models are critically reviewed to identify desirable features to be used in the intended model. Next, the model is derived and implemented into a computational algorithm. This is followed by solving two example problems to explain its efficacy and applicability. Finally, the research work is summarised and directions for further research are introduced.

## CAUSES OF THE FUZZY CONSERVATIVE NATURE

Three related sources for this conservative nature results can be identified. First, all possible combinations of parameter values are considered in fuzzy calculations; while scenarios that combine low probability values in MCS have all the less chance of being randomly selected (Guyonnet *et al.*, 1999). This results in a worst-case scenario, which may be a disadvantage in cases where knowledge of correlations among uncertainties could be used to tighten the bounds on uncertainty (Ferson *et al.*, 1997).

Secondly, the procedures used to implement fuzzy calculations may have an effect. Almost all these procedures are based on interval arithmetic which may result in not only the natural uncertainties, which are directly induced by the uncertainties in the model parameters, but also additional, artificial uncertainties due to the effect of overestimation of interval operations. This effect results from the treatment of fuzzy numbers representing uncertain input parameters as independent numbers, which is not always the case. To tackle or reduce this effect, several approaches have been proposed (e.g. Dong and Shah, 1987; Hanss, 2002). In the vertex method (Dong and Shah, 1987), the $\alpha$-cut concept is employed to discretise a fuzzy number into a number of level sets or intervals. At each level, 2n evaluations are carried out for an n-dimensional function. The transformation method proposed by Hanss (2002) may be considered as an extended version of the vertex method.

The third source of fuzzy conservatism is the dependency of standard fuzzy arithmetic on what are represented by the fuzzy numbers involved in contrary to arithmetic operations on real numbers that follow unique rules. Klir (1997) showed how standard fuzzy arithmetic does not take into account constraints imposed by the meaning of variables. When these constraints, referred to as requisite constraints are neglected the obtained results are less precise than necessary. This is because these constraints represent additional information that should be taken into account. For example, when a variable is known to always equal another variable, an 'equality constraint' should be imposed on standard interval equations because that variable cannot have simultaneously two distinct values. Another constraint is when a variable is known to equal a ratio (crisp or fuzzy) of another variable. These situations may occur in WLC modelling when follow-on costs of competing alternatives are represented as ratios of their initial costs. Obviously, these constraints may easily be imposed by the vertex method if the WLC mathematical model is reformulated to suit a specific problem of interest. A more practical solution is to derive WLC models that can effectively handle these 'normalised costs' without the need to reformulate the model at each application. The objective of the research work underpinning this paper is to develop such a model and to assess its ability to reduce the artificial uncertainties in the calculated whole-life costs.

# A NORMALISED WLC MODEL

Due to its desired features, a normalized version of the NPV model proposed by Kishk and Al-Hajj (2000) will be developed.

$$NPV_i = I_{0i} + \sum_{m=1}^{nno_i} PWO_{im} F_{im} + PWA \sum_{j=1}^{nar_i} A_{ij} + \sum_{k=1}^{nnr_i} PWN_{ik} C_{ik} - PWO_T \cdot SAV_i \tag{1}$$

where $C_{ik}$ are non-annual recurring costs, and $PWN_{ik}$ are their present worth factors given by

$$PWN_{ik} = \frac{1-(1+r)^{-n_{ik} f_{ik}}}{(1+r)^{f_{ik}} - 1} \tag{2}$$

The numbers of occurrences of non-annual recurring costs, $n_{ik}$, are a function of both the analysis period and the frequencies of these costs as follows

$$n_{ik} = \begin{cases} \mathrm{int}(\dfrac{T}{f_{ik}}), & \text{provided that rem}\left(\dfrac{T}{f_{ik}}\right) \neq 0 \\ \dfrac{T}{f_{ik}} - 1, & \text{elsewhere} \end{cases} \tag{3}$$

This model may be rewritten as

$$NPV_i = I_{0i}\left(1 + \sum_{m=1}^{nno_i} PWO_{im} \overline{F}_{im} + PWA \sum_{j=1}^{nar_i} \overline{A}_{ij} + \sum_{k=1}^{nnr_i} PWN_{ik} \overline{C}_{ik} - PWO_T \cdot S\overline{A}V_i\right) \tag{4}$$

where $\overline{F}_{im}$, $\overline{A}_{ij}$, $\overline{C}_{ik}$ and $S\overline{A}V_i$ are normalised variables given by

$$I_{0i} \cdot \overline{F}_{im} = F_{im} \tag{5a}$$

$$I_{0i} \cdot \overline{A}_{ij} = A_{ij} \tag{5b}$$

$$I_{0i} \cdot \overline{C}_{ik} = C_{ik} \tag{5c}$$

$$I_{0i} \cdot S\overline{A}V_i = S\overline{A}V_i \tag{5d}$$

In a similar fashion, a normalised net present value, $N\overline{P}V_i$, may be defined as

$$I_{0i} \cdot N\overline{P}V_i = NPV_i \tag{6}$$

Comparing equations (4) and (5), $N\overline{P}V_i$ can be obtained as

$$N\overline{P}V_i = 1 + \sum_{m=1}^{nno_i} PWO_{im}\overline{F}_{im} + PWA\sum_{j=1}^{nar_i}\overline{A}_{ij} + \sum_{k=1}^{nnr_i} PWN_{ik}\overline{C}_{ik} - PWS \cdot S\overline{A}V_i \tag{7}$$

This normalised value may be seen as an amplification factor applied to the initial cost of an alternative to obtain the whole-life cycle costs of that alternative. Despite this appealing interpretation, $N\overline{P}V_i$ can't be used directly to rank alternatives. Rather, it should be modified such that the reference cost is the same for all alternatives. One way to do so is to choose the initial cost for alternative 1, $I_{01}$. Define a normalised initial cost factor in relation to $I_{01}$, $\hat{I}_i$, as

$$I_{01} \cdot \hat{I}_i = I_{0i} \tag{8}$$

Similarly, define a normalised net present value in relation to $I_{01}$, $N\hat{P}V_i$, as

$$I_{01} \cdot N\hat{P}V_i = NPV_i \tag{9}$$

Substituting from equations (8 and 9) in equation (6) and simplifying, yields

$$N\hat{P}V_i = \hat{I}_i \cdot N\overline{P}V_i \tag{10}$$

Substituting from equations (10) in equation (7) and simplifying, $N\hat{P}V_i$ can be expressed as

$$N\hat{P}V_i = \hat{I}_i\left(1 + \sum_{m=1}^{nno_i} PWO_{im}\overline{F}_{im} + PWA\sum_{j=1}^{nar_i}\overline{A}_{ij} + \sum_{k=1}^{nnr_i} PWN_{ik}\overline{C}_{ik} - PWS \cdot S\overline{A}V_i\right) \tag{11}$$

The model has been implemented into a computer algorithm using the MATLAB® programming environment (The MathWorks, 2000).

## CASE STUDIES

In this section, two example cases are solved using the proposed model to illustrate its effect in improving the conservative nature of WLC fuzzy simulation.

**Case (1)**

A construction firm is considering the purchase of a high-technology equipment. Two systems A and B have been identified and it is required to determine the best system for a discount rate of 4%. The low, best and high estimates of the initial cost of system A are £90,000, £100,000 and £110,000, respectively, with annual running and maintenance costs ranging from 10 to 12% of the initial cost. On the other hand, the initial cost of system B is as twice as that of system A with annual running and maintenance costs ranging from 2% to 3% of its initial costs. The life of either system is 12 years with a negligible salvage value. Figure 1 shows the membership functions (MFs) of cost data in a normalised format.

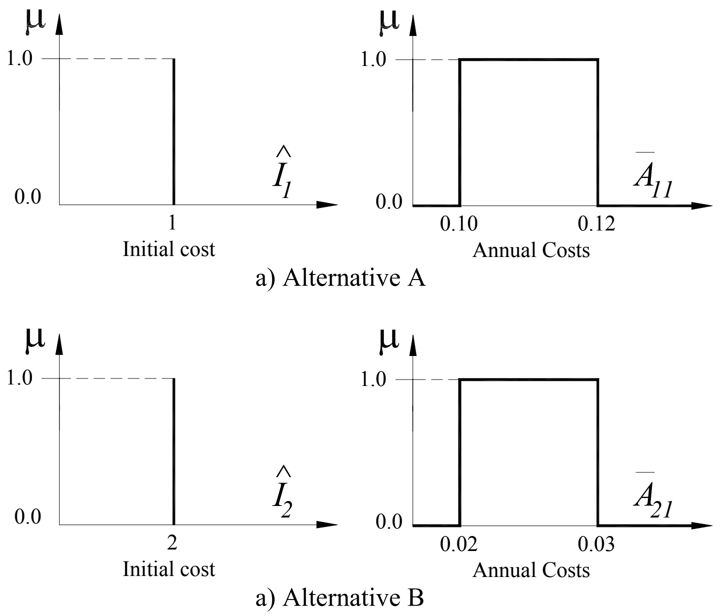


**Figure 1:** Normalized cost data for case (1)

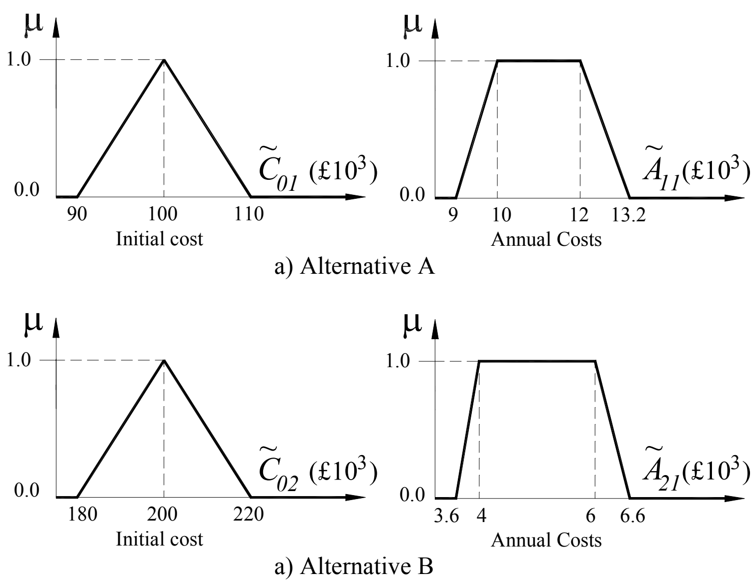


**Figure 2:** Cost data for case (1)

Cost data have also been calculated in a standard format (figure 2) so that the standard NPV model can be used. Figure 3 depicts the resulting NPVs of the competing alternatives in this case. As expected, these MFs are trapezoidal because all cost data are either triangular or trapezoidal and the discount rate and analysis period are certain. The removals for alternatives A and B, marked with down arrows in figure 3, are £203,705 and £247,395, respectively. The confidence measures in this ranking are summarised in Table 1. The uncertainty measures for various discounted cost items have been also calculated as shown in Table 2. As shown, all cost items contributed to the fuzziness of the resulting NPVs.

Because data are normalised, the normalised NPV model can be used directly to solve this example problem. Figure (4) depicts the resulting normalised NPVs of the competing alternatives. As shown, these MFs are rectangular. The removals for alternatives A and B, marked with down arrows in figure (4), are 2.032 and 2.469, respectively. The confidence measures in this ranking were also calculated and are summarised in Table (3). These results reveal that alternative A has an absolute advantage over alternative B.

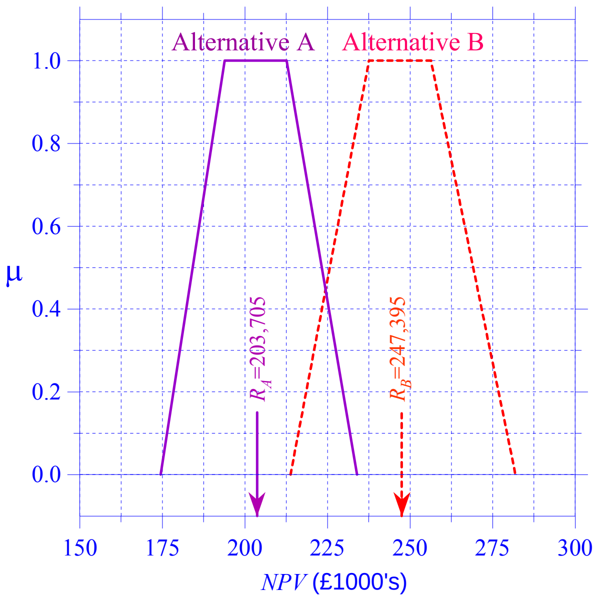


**Figure 3:** Net Present Values of competing alternatives (case 1).

**Table 1:** Measures of confidence in ranking using the NPV model

| Rank | Alternatives | Alternative A | | Alternative B | |
|---|---|---|---|---|---|
| | | $CI_1$ | $CI_2$ | $CI_1$ | $CI_2$ |
| 1 | Alternative A | --- | --- | 0.891 | 0.946 |
| 2 | Alternative B | 0.000 | 0.054 | --- | --- |

**Table 2:** Measures of uncertainty of present values of various costs

| Cost items | Alternative A | | Alternative B | |
|---|---|---|---|---|
| | U | F | U | F |
| Initial cost | 2.197 | 10.000 | 2.807 | 20.000 |
| Annual costs | 3.384 | 10.324 | 3.191 | 4.693 |

**Table 3:** Measures of confidence in ranking using the normalized model

| Rank | Alternatives | Alternative A | | Alternative B | |
|---|---|---|---|---|---|
| | | CI1 | CI2 | CI1 | CI2 |
| 1 | Alternative A | --- | --- | 1.000 | 1.000 |
| 2 | Alternative B | 0.000 | 0.000 | --- | --- |

This superior performance of the normalised NPV algorithm over the standard NPV algorithm can be attributed to the elimination of fuzziness in cost data through the normalisation process as shown in figure 2 where all normalised input data are represented by spikes and trapezoidal MFs. Thus, there is no fuzziness of all NPV contributions as shown in Table (4). Another unique feature of the normalised model can be noticed. Both the non-specificity and fuzziness measures of the initial cost of alternative B are also zero. This point is further discussed in the next subsection.

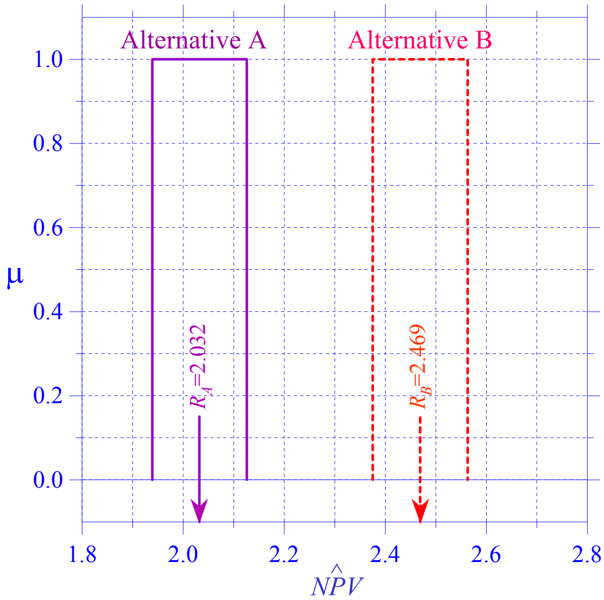


**Figure 4**: Normalised net present values of competing alternatives (case 1).

**Table 4** Measures of uncertainty of normalised present values of various costs

| Cost items (discounted & normalised) | Alternative A | | Alternative B | |
|---|---|---|---|---|
| | U | F | U | F |
| Initial cost | 0.000 | 0.000 | 0.000 | 0.000 |
| Annual costs | 0.172 | 0.000 | 0.172 | 0.000 |

**Case (2)**

The effect of using fuzzy normalised data on the performance of the normalised model is investigated in this section. The new normalised data are shown in figure (5) and all other data are the same as in case study 1. The NPV algorithm (Kishk and Al-Hajj, 2000) was employed to solve this example problem. Figure (6) depicts the resulting NPVs of the competing alternatives. The removals for alternatives A and B are £218,174 and £231,7055, respectively, indicating that alternative A outranks alternative B. The confidence measures in this ranking are summarised in Table (5). As expected, these measures are smaller than those in the non-fuzzy case (Table 1).

**Table 5:** Measures of confidence in ranking using the NPV model (case 2)

| Rank | Alternatives | Alternative A | | Alternative B | |
|---|---|---|---|---|---|
| | | CI1 | CI2 | CI1 | CI2 |
| 1 | Alternative A | --- | --- | 0.252 | 0.596 |
| 2 | Alternative B | 0.060 | 0.404 | --- | --- |

The uncertainty measures for present worth contributions of various cost items have been also calculated and are shown in Table (6). Obviously, the uncertainty measures of the initial costs of alternative A are the same as in the non-fuzzy case (Table 2). However, all other cost items contributed more fuzziness to the resulting NPVs than in the non-fuzzy case (Table 2). This illustrates the lower confidence measures obtained.

**Table 6:** Measures of uncertainty of present worths of various costs (case 2)

| Cost items | Alternative A | | Alternative B | |
|---|---|---|---|---|
| | U | F | U | F |
| Initial cost | 2.197 | 10.000 | 3.741 | 53.500 |
| Annual costs | 3.596 | 30.502 | 3.127 | 27.692 |

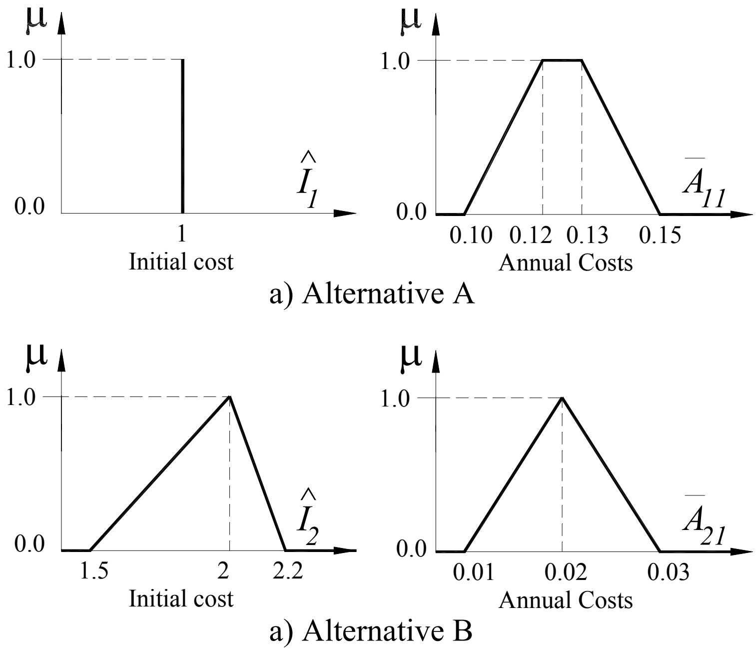


**Figure 5**: Normalised cost data for case (2)

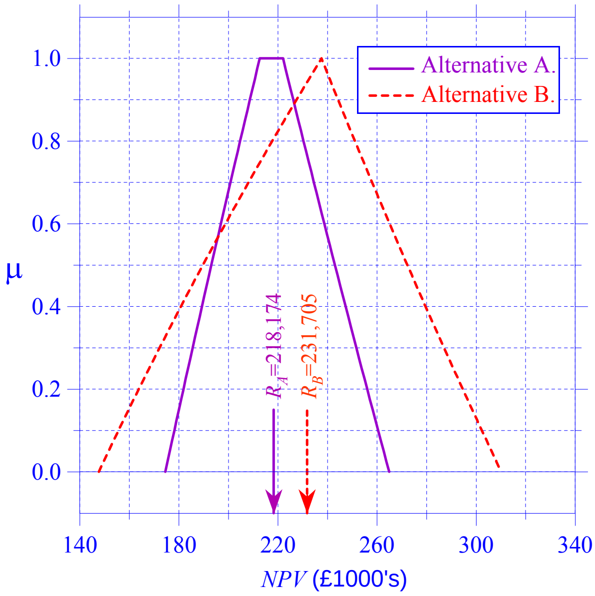


**Figure 6:** Net Present Values of competing alternatives ( case 2)

The normalised NPV algorithm was also employed to solve this example problem. Figure (7) depicts the resulting normalised NPVs of the competing alternatives. The removals for alternatives A and B, marked with down arrows in figure 7, are 2.173 and 2.297, respectively, indicating again that alternative A outranks alternative B. The confidence measures in this ranking were also calculated and are summarised in table (7). As expected, these measures are smaller than those in the non-fuzzy case (table 4). However, they are higher than those obtained by the NPV algorithm. This indicates again the superiority of the normalised model over the standard NPV model.

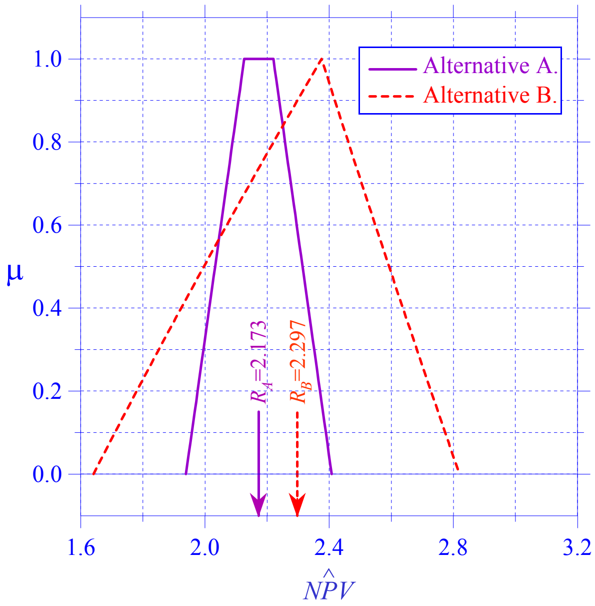


**Figure 7:** Normalized Net Present Values of alternatives of case (2)

**Table 7:** Measures of confidence using the normalised NPV model (case 2)

| Rank | Alternatives | Alternative A | | Alternative B | |
|---|---|---|---|---|---|
| | | CI1 | CI2 | CI1 | CI2 |
| 1 | Alternative A | --- | --- | 0.353 | 0.627 |
| 2 | Alternative B | 0.098 | 0.373 | --- | --- |

The uncertainty measures for various discounted cost items have been also calculated and are shown in Table (8). As shown, all normalised cost items contributed more fuzziness to the resulting NPVs than in the non-fuzzy case (Table 4). However, the uncertainty measures of the initial costs of alternative A (used in the normalisation process) are still zero. This illustrates again that the superiority of the proposed model is due to the elimination of expanding normalised ratios to the standard format.

**Table 8:** Measures of uncertainty of normalized present values of various costs (case 2)

| Cost items | Alternative A | | Alternative B | |
|---|---|---|---|---|
| | U | F | U | F |
| Initial cost | 0.000 | 0.000 | 0.289 | 0.350 |
| Annual costs | 0.245 | 0.188 | 0.212 | 0.239 |

## CONCLUSIONS AND THE WAY FORWARD

Results obtained from fuzzy WLC models are more conservative than probabilistic counterparts. The main cause of this conservative nature is that standard fuzzy arithmetic ignores existing correlations among input uncertainties. Besides, its dependency on what are represented by the fuzzy numbers involved. These situations may occur in WLC modelling when follow-on costs of competing alternatives are represented as ratios of their initial costs.

To deal with these situations, a novel normalized WLC model has been derived such that uncertainty of all input variables can be effectively modelled. Besides, the resulting whole-life costs are ratios of initial costs with a clear interpretation. The applicability of the model has been illustrated in the context of two example applications. Typical results showed its advantage over the standard NPV model when dealing with normalized data. First, the model saves the time of preparing the data in the standard format. Secondly, and more importantly, the confidence measures in ranking can be better because of the elimination of the additional uncertainty in the predicted WLCs that may be caused by expanding normalised data to the standard format.

Handling the conservative nature of fuzzy simulations of statistically significant data represented by probability distribution functions is being investigated and will be reported in a future paper.

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