

EXPLORATION OF RANKING METHOD OF GROUP DECISION MAKING

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Ordinal summing method is the most popular ranking method of group decision making. When using this method, the final ranking of projects is determined by summing up the ordinals given by individual decision makers. Therefore, the individual decision makers' evaluation is the key factor affecting the final group-ranking result of each project. However, since it is not possible to effectively distinguish the degree of difference in evaluation results among individual decision makers, the deviated evaluation behaviour of the individual decision makers, if any, will be difficult to discover, and the final ranking is easy to be affected by few decision makers' extreme evaluations. This research proposes an improved ranking procedure based on utilizing fuzzy relation matrix and eigenvector method that conform to human judgments. Through the validation of a simulated case, the improved procedure proposed in this research can actually examine the consistency of group ranking and the difference of individual decision makers' evaluations through a rigorous quantitative method. It effectively improves the disadvantages of group decision making and greatly upgrades the quality of decision making.

Keywords: fuzzy relation, group decision making, ranking method.

INTRODUCTION

Introduction of group decision making

Project evaluation is a considerably prevalent subject of decision making in government and enterprise. Due to the increasing complexity and uncertainty of subjects, the decision-making process needs to be more comprehensive. Nowadays, group decision making becomes more widely applied. Using the government procurement in Taiwan as an example, in 2005, the contracts of 2.65 billion dollars in total were awarded to tenders by the method of group decision making (Wang and Tsai 2006). Therefore, the group decision making process is worthy further research and investigation.

It is called "group decision making" if the decision maker refers to a decision group or committee. Stephn (1992) indicates that during the process of decision making, group is a significant tool. If the group is formed by people with different backgrounds, there might be more substitute projects and the analysis will be more precise.

According to Teng's review (2002), he points out that the policy of group decision making has disadvantages such as time wasted in making decision and uncertainty of

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responsibilities, though it has the merits of beneficial mind-gathering and avoids personal subjective deviations (as see Table 1).

Table 1: Comparison of advantages and disadvantages of group decision making (Teng 2002)

Advantages	<ol style="list-style-type: none"> 1. To draw on collective wisdom and absorb all useful ideas. 2. Increasing creativity. 3. Facilitating participation, and understanding. 4. Minimizing personal subjectivity.
Disadvantages	<ol style="list-style-type: none"> 1. Risk-avoidance phenomena. 2. Controlled by minor group of people. 3. Uncertainty in responsibilities. 4. Emphasis on trivial things. 5. Time consuming and higher costs.

Types of preference integrations in group decision making

When using group decision making to manage project evaluation, the decision makers with different professional backgrounds might have different evaluations. In order to acquire acceptable compromise solutions or trade-off results, the preference of individual decision makers should be integrated. Generally speaking, preference integration can be divided into two types - pool first and pool last (Buckley 1985).

1. Pool first

The evaluations of R decision makers are first integrated into a single score at different evaluation principles, and then the multiple criteria evaluation method is used to compare and rank n projects.

2. Pool last

First, R decision makers use the multiple criteria evaluation method to compare and rank n projects. All decision makers' preferences are then integrated to obtain the final ranking of group decision making.

This research mainly focuses on the pool last method to explore the possible improvement in looking for the final ranking of group decision making. We initially analyze the drawbacks of the ordinal summing method which is the most popular ranking method of group decision making, and then propose an improved ranking procedure. This research further validates this proposed procedure with simulated cases to examine its advantages and feasibility in improving the quality of group decision making.

ORDINAL SUMMING METHOD

Among the methods of group decision making, ordinal summing method is most widely used. For instance, in Taiwan, this method is used to award contract in 73.9% of government procurement cases of the most advantageous tender in 2003 (Tsai and Wang 2005). In this section, we will briefly introduce the ordinal summing method, and discuss its drawbacks.

Major steps

Since ordinal summing method is simple and easy to understand, it is more popular in empirical uses than other methods. The method is mainly divided into two major steps.

First, each decision maker determines the rank for each of the evaluated projects according to their performance. The smaller value of the rank represents the better performance of the project.

Second, the acquired ranks are summed up for each project. After rearranging the projects based on the total ordinal from the lowest to the highest, the result is the final ranking of group decision making.

For example, Table 2 illustrates the evaluation results of three projects *a*, *b*, and *c* by three decision makers *A*, *B*, and *C*. After adding the ordinal acquired by each project and rearranging the projects from the lowest to the highest summed ordinal, the final ranking of this group decision making is { $b \succ a \succ c$ }.

Table 2: Summary of evaluation results

Project Decision maker	<i>a</i>	<i>b</i>	<i>c</i>
<i>A</i>	2	1	3
<i>B</i>	1	2	3
<i>C</i>	2	1	3
Total	5	4	9
Ranking	2	1	3

Drawbacks

From the aforementioned discussion, it reveals that ordinal summing method mainly sums up the ordinals from all decision members to decide the final ranking of the evaluated projects. However, this method obviously encounters several drawbacks.

1. It is against the fundamental principles of measurement.

Ordinal summing method measures the overall performance of projects by ranking, which is a kind of ordinal scale in qualitative scaling. Such an ordinal scale allows ranking the projects, but it is not applicable to mathematical calculation since the differences between ranks are not equal and no multiple relationships exist among ranks (Chow 2004). This ordinal summing method, however, sums up the ordinals for each project and treats the project with the lowest summed ordinal as the best project. In doing, it apparently conflicts with the fundamental principles of measurement.

2. It is easy to be affected by few decision makers' extreme evaluations.

Ordinal summing method works by summing up the ranks for each project. If a few of the decision makers tend to give the best or the worst ordinal to a particular project, their extreme opinions may twist the final ranking result as well as affect the quality of the overall group decision making.

3. It is hard to distinguish the differences in the decision makers' evaluation.

Ordinal summing method does not compare the differences in the decision makers' evaluation during the process. When the extreme evaluation by few decision makers may affect the final ranking result, as mentioned previously, this method lacks an effective strategy to distinguish the differences in evaluation.

How to integrate the evaluation of individual decision makers into an acceptable ranking result representing the whole group remains an issue that draws much research attention. Several methods have been proposed to primarily prevent the group ranking decision from being influenced by few decision makers' biased evaluations, such as Borda count method (Roberts 1976, Goddard 1983), consensus ranking method (Cook and Seiford 1978), and fuzzy preference relation ranking method (Lu 2004). These methods, however, do not provide effective strategies to determine the consistency of group ranking result nor identify the biased evaluation of individual decision makers.

Pre-requisites of an ideal group ranking procedure

This research suggests that an ideal ranking procedure of group decision making should have the following characteristics.

1. Based on a theoretical foundation.

For any evaluation systems, the methods of measuring performance should base on theoretical principles. Otherwise, the results derived may not hold practical significances.

2. Examining the consistency in group ranking result.

In order to obtain representative evaluation results as well as to prevent the final ranking from being influenced by the extreme evaluation of few decision makers, this research recommends that the examination of consistency in group ranking is needed. That is, a reasonable group ranking result should fulfil the requirements of transitivity.

3. Being capable of identifying extreme evaluation of individual decision makers.

If the group ranking result fails to meet the requirement of consistency, it means that the evaluation result might be biased by extreme evaluations of few decision makers. An ideal ranking method should be able to identify the extreme evaluation of few decision makers.

AN IMPROVED RANKING PROCEDURE OF GROUP DECISION MAKING

Based on the aforementioned pre-requisites of an ideal group ranking procedure, this research proposes an improved ranking procedure of group decision making. The improved procedure is summarized in a flowchart (see Figure 1) and the steps are described respectively as following.

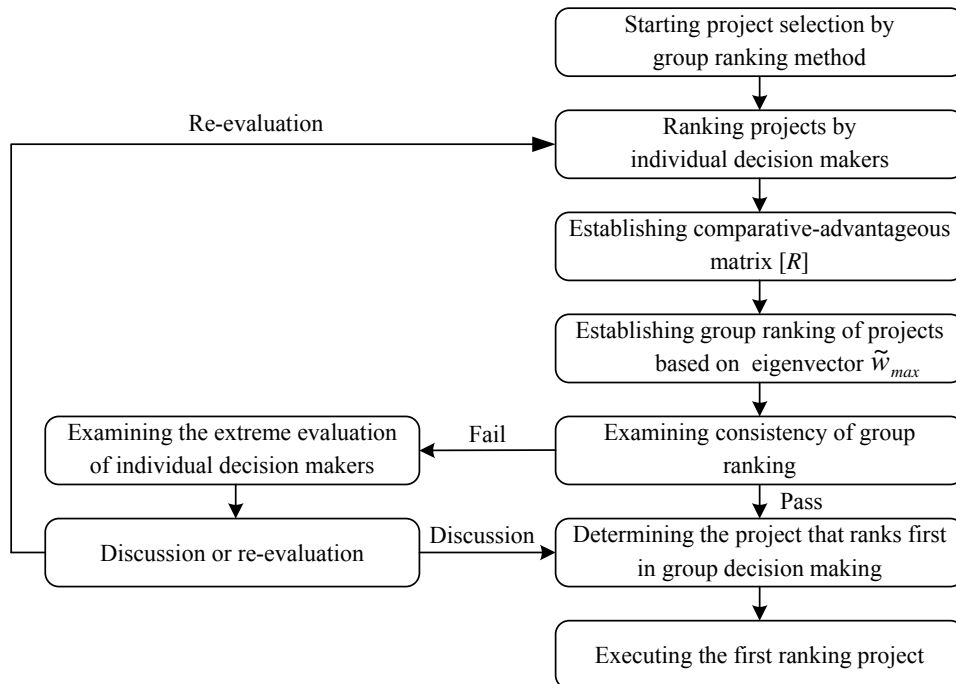


Figure 1: Flow chart of improved group decision making ranking method.

Establishing comparative-advantageous (CA) matrix [R]

According to the process of ordinal summing method, it arbitrarily implies that the first-ranking project is twice as good as the second, three times as good as the third, and so forth. In fact, the ranking order given by the decision makers only shows the order of preference. That is, the first-ranking is “more superior” than the second which is “more superior” than the third, but no clear levels of differences among rankings are stated. Therefore, this ranking is an expression of a type of fuzzy relationship (Lu 2004). Based on the method proposed by Seo and Sakawa (1985), this research uses the ratio of people in their preference between two projects to represent the comparative-advantageous degree. The fuzzy relationship can be used to create a comparative-advantageous (CA) matrix [R].

For instance, N decision makers evaluate k projects. Upon completion of ranking, a CA matrix [R] can be established based on the evaluation of each decision maker.

$$CA\ matrix\ [R] = \begin{matrix} & X_1 & X_2 & \cdots & X_k \\ \begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{matrix} & \begin{bmatrix} 1 & r_{12} & \cdots & r_{1k} \\ r_{21} & 1 & \cdots & r_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ r_{k1} & r_{k2} & \cdots & 1 \end{bmatrix} \end{matrix} \quad (1)$$

In this matrix, $r_{ij} = \frac{n_{ij}}{N}$, and it refers to the comparative-advantageous degree of project X_i toward project X_j . n_{ij} denotes the number of decision makers who believe project X_i is better than project X_j .

The elements on the diagonal, r_{ii} , show the comparisons of projects to themselves. Therefore, they are equal to 1.

Establishing group ranking of projects based on eigenvector \tilde{w}_{max}

It has been known that N eigenvalues and N eigenvectors can be attained from any N -rank square matrix [A] (Cheng 1995). According to the aforementioned characteristics of a square matrix, this research adopts eigenvectors to establish the final ranking of group decision making.

After constructing the comparative-advantageous matrix [R] based on individual decision makers’ evaluations, this study uses $[R]\tilde{w} = \lambda\tilde{w}$ to attain the eigenvalue λ and eigenvector \tilde{w} . The maximum eigenvector \tilde{w}_{max} corresponding to the maximum eigenvalue λ_{max} is obtained to represent the overall advantageous relationship of each project. The magnitude of absolute values of elements in the eigenvector \tilde{w}_{max} serves as a reference for decision makers when they perform group ranking of all projects.

The higher the consistency among decision makers in their evaluation of advantageousness of a project, the greater the absolute value of the corresponding element in eigenvector \tilde{w}_{max} for that particular project. In contrast, the lower the consistency among decision makers, the smaller the absolute value of the

corresponding element in eigenvector \tilde{w}_{max} for that particular project. The absolute value of elements in eigenvector \tilde{w}_{max} has an upper limit of 1 and a lower limit of 0.

Examining consistency of group ranking

If r_{ij} is larger than 0.5, that means project i is more superior to project j . However, if a small number of decision makers give extremely biased evaluations to a particular project (such as giving a worse ranking to project i), the final ranking of projects is likely to be influenced. Therefore, this research recommends the following procedure to examine the consistency of group ranking.

1. The proposed procedure starts with resorting the columns and rows of the CA matrix $[R]$ from the lowest to the highest ordinal of group ranking, and further acquires a rearranged-comparative-advantageous (RCA) matrix $[\hat{R}]$.
2. The next step is to examine the consistency based on the upper-triangular elements in the matrix $[\hat{R}]$. Since the elements in matrix $[\hat{R}]$ are arranged based on the result of group ranking, if the requirement of consistency is met, the upper-triangular elements \hat{r}_{ij} should be larger than 0.5 to reflect that project i has better ordinal in group ranking than project j . On the contrary, if there are some upper-triangular elements \hat{r}_{ij} smaller than 0.5, it means that the group ranking is inconsistent. That is, although project i has better ordinal in group ranking than project j , in fact, more than half of the decision makers believe project j to be better than project i .

If the group ranking meets the requirement of consistency, this research suggests that the first-ranking project can be determined by eigenvector \tilde{w}_{max} . In contrast, if it fails to meet the requirement, it means that the group ranking result is biased by extreme evaluations of few decision makers. In this case, examining whether there is any extreme evaluation of individual decision makers is needed.

Examining the extreme evaluation of individual decision makers

When the results of group ranking are inconsistent, it means that some decision makers offer extreme evaluations for certain specific projects. This research recommends the following steps to examine whether individual decision makers show obviously deviated evaluation results.

1. Constructing an individual evaluation (IE) matrix $[P^n]$ for each decision maker.

For example, if there are k projects for evaluation, the IE matrix $[P^n]$ of decision maker n is:

$$IE \text{ matrix of member } n [P^n] = \begin{matrix} & X_1 & X_2 & \cdots & X_k \\ X_1 & \begin{bmatrix} 1 & p_{12}^n & \cdots & p_{1k}^n \\ p_{21}^n & 1 & \cdots & p_{2k}^n \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1}^n & p_{k2}^n & \cdots & 1 \end{bmatrix} & & & \end{matrix} \quad (2)$$

$$p_{ii}^n = 1, \quad p_{ij}^n = 0 \text{ or } 1 \quad \text{for all } i \neq j$$

Where elements p_{ij}^n and p_{ji}^n show the judgment of decision maker n for project i and project j . If project i is better than project j , $p_{ij}^n = 1$ and $p_{ji}^n = 0$. Conversely, if project j is better than project i , $p_{ij}^n = 0$ and $p_{ji}^n = 1$.

2. Constructing an individual evaluation-difference (IED) matrix $[E^n]$ for each decision maker based on the difference between $[P^n]$ and $[R]$.

The IED matrix $[E^n]$ signifies the differences of evaluation results between individual decision maker n and the whole group.

$$[E^n] = [P^n] - [R] \quad (3)$$

Where element $e_{ij}^n = p_{ij}^n - r_{ij}$

Element e_{ij}^n in matrix $[E^n]$ means the degree of differences in evaluation results for project i and project j between decision maker n and the whole group. Value of e_{ij}^n ranges between -1 and 1; thus, the absolute value $|e_{ij}^n|$ ranges between 0 and 1. The greater the absolute value, the greater the difference.

3. Determining extreme evaluation of individual decision maker based on the elements in the IED matrix $[E^n]$.

This research suggests that, in $[E^n]$ matrix, if more than half of elements in the same row with value over 0.5 or less than -0.5, it means that this decision maker differs from more than half of other decision makers regarding the evaluation of that particular project, and the differences in ranking have reached more than 50% of the whole ranking. In other words, this decision maker has displayed a certain degree of deviation in ranking. For this situation, we suggest that the all decision makers should discuss this situation and explore the possible solutions or even consider the necessity of re-evaluation.

CASE SIMULATION

In this simulated case, it is assumed that there are five projects a, b, c, d, e for evaluation and 7 decision makers A, B, C and G . The evaluation results are summarized in Table 3.

Table 3: Original evaluation results of decision makers

Project Decision maker	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>A</i>	4	3	1	2	5
<i>B</i>	2	4	1	3	5
<i>C</i>	1	4	3	2	5
<i>D</i>	3	2	4	1	5
<i>E</i>	3	5	1	2	4
<i>F</i>	3	4	5	1	2
<i>G</i>	3	4	1	2	5

Establishing comparative-advantageous matrix [R]

According to the improved procedure proposed in this research, a CA matrix [R] is first established.

$$CA\ matrix\ [R] = \begin{matrix} & a & b & c & d & e \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 1.00 & 0.71 & 0.43 & 0.29 & 0.86 \\ 0.29 & 1.00 & 0.29 & 0.00 & 0.71 \\ 0.57 & 0.71 & 1.00 & 0.57 & 0.86 \\ 0.71 & 1.00 & 0.43 & 1.00 & 1.00 \\ 0.14 & 0.29 & 0.14 & 0.00 & 1.00 \end{bmatrix} \end{matrix} \quad (4)$$

The elements in the matrix are defined as $r_{ij} = \frac{n_{ij}}{7}$, where n_{ij} represents the number of decision makers who believe project i is better than project j .

Determining group ranking of projects based on eigenvector \tilde{w}_{max}

Eigenvector method is used to determine the group ranking result of projects. After calculating, the absolute value of the maximum eigenvalue $|\lambda_{max}|$ in [R] is 2.58, and the corresponding eigenvector $\tilde{w}_{max} = \{ 0.46, 0.25, 0.58, 0.61, 0.14 \}$. This signifies that the group ranking of projects given by decision makers is $\{ d > c > a > b > e \}$.

Examining the consistency of group ranking

A RCA matrix $[\hat{R}]$ is obtained by resorting the columns and rows of the CA matrix [R] by group ranking from the lowest to the highest values.

$$RCA\ matrix\ [\hat{R}] = \begin{matrix} & d & c & a & b & e \\ \begin{matrix} d \\ c \\ a \\ b \\ e \end{matrix} & \begin{bmatrix} 1.00 & 0.43 & 0.71 & 1.00 & 1.00 \\ 0.57 & 1.00 & 0.57 & 0.71 & 0.86 \\ 0.29 & 0.43 & 1.00 & 0.71 & 0.86 \\ 0.00 & 0.29 & 0.29 & 1.00 & 0.71 \\ 0.00 & 0.14 & 0.14 & 0.29 & 1.00 \end{bmatrix} \end{matrix} \quad (5)$$

In the matrix $[\hat{R}]$, one upper-triangular element $\hat{r}_{dc} = 0.43$ is smaller than 0.5. It means that, although project d is better than project c in the final result of group ranking, only 43% of decision makers believe that project d is better than project c . That is, there are 57% of decision makers think that project c is better than project d . Thus, the result of group ranking acquired in the previous step does not match the requirement of consistency. It means that some decision makers offer extremely deviated ranking for specific projects. Since these deviated evaluation behaviours affect the consistency of group ranking, it is necessary to further examine the evaluation difference of individual decision makers.

Examining the extreme evaluation of individual decision makers

In order to examine the extreme evaluation of individual decision makes, the next step is to construct an IE matrix $[P^n]$ and an IED matrix $[E^n]$ for each decision maker. After checking the IED matrix of each decision maker, we can find that some rows in matrices $[E^D]$ and $[E^F]$ having elements with values less than -0.5 or larger than 0.5.

$$\begin{matrix} & & a & b & c & d & e \\ & a & 0.00 & -0.71 & 0.57 & -0.29 & 0.14 \\ & b & 0.71 & 0.00 & 0.71 & 0.00 & 0.29 \\ \text{IED matrix of member } D & [E^D] = c & \underline{-0.57} & \underline{-0.71} & 0.00 & \underline{-0.57} & 0.14 \\ & d & 0.29 & 0.00 & 0.57 & 0.00 & 0.00 \\ & e & -0.14 & -0.29 & -0.14 & 0.00 & 0.00 \end{matrix} \quad (6)$$

In matrix $[E^D]$, there are more than half elements in row c with values less than -0.5 ($e_{ca}^D = -0.57$, $e_{cb}^D = -0.71$, $e_{cd}^D = -0.57$). It means that the results of pair-wise comparisons between projects c and a , projects c and b , and projects c and d for decision maker D are different from those of more than half of the decision makers. In other words, there are over half of decision makers believe that project c is better than projects a , b and d , while decision maker D 's evaluations are in the opposite direction. Comparing with other decision makers, decision maker D 's evaluation toward project c is significantly different.

$$\begin{matrix} & & a & b & c & d & e \\ & a & 0.00 & 0.29 & 0.57 & -0.29 & -0.86 \\ & b & -0.29 & 0.00 & 0.71 & 0.00 & -0.71 \\ \text{IED matrix of member } F & [E^F] = c & \underline{-0.57} & \underline{-0.71} & 0.00 & \underline{-0.57} & \underline{-0.86} \\ & d & 0.29 & 0.00 & 0.57 & 0.00 & 0.00 \\ & e & \underline{0.86} & \underline{0.71} & \underline{0.86} & 0.00 & 0.00 \end{matrix} \quad (7)$$

In matrix $[E^F]$, four elements in row c are smaller than -0.5 ($e_{ca}^F = -0.57$, $e_{cb}^F = -0.71$, $e_{cd}^F = -0.57$, $e_{ce}^F = -0.86$). It means that results of decision maker F 's pair-wise comparisons between projects c and a , projects c and b , projects c and d , and projects c and e are different from those of the majority of other decision makers. In other words, there are over half of decision makers believe that project c is better than projects a , b , d and e . Decision maker F 's evaluations, however, provide the contrary result.

In addition, three elements in row e are greater than 0.5 ($e_{ea}^F = 0.86$, $e_{eb}^F = 0.71$, $e_{ec}^F = 0.86$). It reveals that the results of decision maker F 's pair-wise comparisons of project e against projects a , b , and c are different from those of over half of the decision makers. That is, there are more than half of decision makers believe that project e does not perform as good as projects a , b , and c , whereas decision maker F gives the opposite evaluation. Comparing with other decision makers, decision maker F 's evaluations toward project c and e are significantly different.

Comparing with the rest of decision makers, decision makers D 's and F 's evaluations toward project c is apparently lower, and decision maker F 's evaluation toward project e is obviously higher. Decision makers D and F should explain their evaluation result, and all of the decision makers may need to further discuss how to deal with such situation, explore the possible solutions, and even consider re-evaluating the projects.

CONCLUSION

When using ranking method to select the best project, how to decide the first-ranking project is the critical determinant of the whole selection process. This research proposes an improved evaluation procedure based on the utilization of fuzzy relation matrix and eigenvector method that conform to human judgments. This improved procedure not only possesses theoretical foundations, but it also enables to examine the consistency in group evaluation results and extreme evaluation of individual decision makers. With the analysis of a simulated case, the result validates that the improved procedure is able to detect and then lower the impacts extreme evaluations of few decision makers. Therefore, this improved ranking procedure may upgrade the quality of group decision making.

REFERENCES

- Buckley, J. J. (1985) Fuzzy hierarchical analysis. *Fuzzy Sets and Systems*, **15**, 21-31.
- Cheng, C. (1995) *Advanced Engineering Mathematics*. Taipei: Wen-Sheng Publishing Co. LTD.
- Chow, W. S. (2004) *Multivariate Statistical Analysis: with Application of SAS/ STAT*. Taipei: Best-Wise publishing Co. LTD.
- Cook, W. and Seiford, L. (1978) Priority ranking and consensus formation. *Management Science*, **24**(16), 1721-32.
- Goddard, S. T. (1983) Ranking in tournaments and group decision making. *Management Science*, **29**(12), 1384-92.
- Lu, S. T. (2004) *Decision-Making Models of Technical Service Suppliers Selection in Government Procurement*, Unpublished PhD Thesis, Department of Civil Engineering, National Central University.
- Roberts, F. S. (1976) *Discrete Mathematical Models*. New Jersey: Prentice-Hall.
- Seo, F. and Sakawa, M. (1985) Fuzzy Multiattribute Utility Analysis for Collective Choice. *IEEE Transactions on Systems, Man, and Cybernetics*, **15**(1), 45-53.
- Stephn, P. R. (1992) *Organizational Behavior*. New Jersey: Prentice Hall.
- Teng, J. Y. (2002) *Project Evaluation: Methods and Applications*. Keelung: National Taiwan Ocean University, Logistics Planning and Management Research Center.
- Tsai, H. Y., and Wang, L. C. (2005) Preliminary discussion on most advantageous tendering implementation. *9th United Construction Engineering and Management Research Presentation*, Taipei.
- Wang, L. C., and Tsai, H. Y. (2006) A study of improving the ranking procedure for determining the most advantageous tender. *Journal of Construction Management (Taiwan)*, **67**(2), 9-18.