AN IMPARTIAL DECISION MODEL FOR FINANCIAL RISK AND DECISION ANALYSIS IN CONSTRUCTION PROJECTS

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In financial decision-making processes, especially in the construction industry, tangible and intangible criteria always co-exist and their weights have significant impact on the final decision outcome. Human cognitive thinking cannot be easily modelled and quantified by rational rules in conventional multi-criterion decision aid (MCDA) methods while uncertainties such as bias of decision-maker who either under or over estimates a criterion should be quantified as it contributes heavily in weight evaluation. Entropy has been useful in quantifying uncertainty in decision-making. This paper illustrates an Impartial Decision Model (IDM) that is entropy-based in three dimensional vectors to the solution of multi-criterion financial decision analysis in construction. A two-dimension plane is formulated as the inclusion of four principal vectors which are the relative weights between sub-criteria (activities); the relative weights between alternatives (projects); the relative weights between criteria (financial risks); and the weights of criteria for each sub-criterion arising on each alternative. Risk adjusted discount rate (RADR) of each vector is incorporated with the weights derived by the two-dimension plane to obtain final decision weights. The financial risks of multi-projects being undertaken by a medium-size construction firm in Hong Kong were assessed to evaluate the model. The results indicated that uncertainties in each vector have been quantified to provide an upper, mean and lower bound financial risks on projects with inconsistencies or total uncertainty level 0.009, 0.032 and 0.036 on all sub-criteria in one alternative respectively. The risk adjusted discount overall cash flow was 0.97 millions more than the original forecasted (20.84 millions). An accurate, objective and realistic decision on financial risk analysis can be provided to the decision-maker to evaluate, select and control the projects by rating impartially and discounting the cash flow in terms of risk rate.

Keywords: construction, decision-making, entropy, multi-criteria analysis, weighting.

INTRODUCTION

Decision-making problem abounds in real world. In financial decision-making processes, especially in the construction industry, tangible and intangible criteria always co-exist and conflicting (Zopounidis 1999; Steuer and Na 2003). The weights of the criteria have significant impacts on the final decision outcome. However, human cognitive thinking cannot be easily modelled and quantified by rational rules. Mapping of the decision maker’s cognition in design management process has been done using a cognitive approach (Edkins, 1998). However, uncertainty such as the bias of a decision-maker who either under or over estimates a criterion should be quantified as it contributes significantly in weight evaluation. The final outcome of a decision might be out of expectation as the decision-maker was uncertain at the time.
of evaluating the criterion like gambling. As a consequence, the amount of uncertainties in the decision-making process causes the final outcome deviated from the possible outcome in a dimension relative to the uncertain level. In the literatures, none of the well-known multi-criterion decision aid (MCDA) methods such as the entropy method (Shannon 1948), the Multiple Attribute Utility Theory (MAUT) (Keeney and Raiffa 1976), the Analytic Hierarchy Process (AHP) (Saaty 1980), and the principal right eigenvector method (Saaty 1988), is capable to cover all decision-making circumstances (Guitouni and Martel 1998; Cho 2003) nor provides uncertainty quantification.

THE PROBLEM DEFINED

Construction finance is a non-commensurable multi-criterion decision-making problem. It allows contracting firms to optimize their resource allocation, plan for financial need, predict profit, and forecast cash flow liquidity at both the project and the corporate levels. Valuable information, such as cash flow diagrams, the amount of capitals required to perform a contract and the amount of interest due for supporting overdrafts can be generated using financial models to support the contractor’s financial decision (Kaka and Price 1991). A number of mathematical financial models for deriving and forecasting the project budgets have been developed (Nazem 1968; Kennedy et al. 1970; Ashley and Teicholz 1977; McCaffer 1979; Peer 1982; Kenley and Wilson 1986; Kaka and Price 1993). Knowing the interaction of criteria in construction finance enhances the accuracy of forecasting. A net cash flow model considering the previously ignored variables such as inflation and retention was developed (Kaka and Price 1991). However, there was no effective way to consider variable interaction which was the main subject domain for sketching the cash flow profiles. It has then been improved by the use of approximate estimates (Skitmore and Marsden 1999), artificial neural network (Boussabaine and Kaka 1998), fuzzy techniques (Boussabaine and Elhag 1999), heuristic method (Son and Skibniewski 1999), adaptive genetic algorithm (AGA) (Lam et al. 2001), dynamic approach at the corporate level (Kaka and Lewis 2003) and practical multifactor approach (Warszawski 2004). These techniques were considered to be helpful to contractors despite the fact that they are based on a number of simple assumptions, such as the linearization of non-linear objective functions and constraints and the neglect of in-depth study of the complex interactions between the criteria concerned. Financial decisions are thus not fully structured, ill-defined and full of uncertainties.

Multi-criterion decision-making problems require the information about the relative importance or weight of criteria in order to choose the best alternative. Therefore, the estimation of the relative weights of criteria plays a significant role in a MCDA method (Eckenrode 1965). Many well-known MCDA methods as mentioned were available for the solution of multi-criterion decision analysis that needed trade-offs. They were best applied in selecting alternatives subject to tangible and intangible multi-criteria and hence useful in the planning stage of the financial analysis in construction. As construction finance forecasting is time-based, the monitoring and control of it is much more significant especially in a highly vague environment. Besides, behavioural influences (Weber and Borcharding 1993) and the decision-makers themselves (Chen and Kane 2001) were indeed the most significant determinants in the analysis, regardless of the MCDA method used. However, none of the conventional MCDA methods as discussed provided uncertainty quantification of
the decision-makers. An impartial environment thus can be beneficial to the planning, monitoring and control of financial analysis in construction.

The research attempted to provide a solution to the problem. The work has the following main objectives: (1) to develop an entropy-based IDM for deriving impartial decision weights in financial decision-making problem; and (2) to demonstrate and evaluate the model using a case study. This provides an accurate, objective, reliable and reasonable basis for financial decision-making process.

THE IMPARTIAL DECISION MODEL (IDM)

The IDM is entropy-based. Fig. 1 shows four main steps of the model to determine the uncertainties in multi-criterion financial evaluation, monitoring and control in the construction industry by a discounting method. Through the IDM, a contingent outcome can be produced such that an early warning can be provided to the decision-maker to take appropriate management strategy prior to the sign of insolvency or bankruptcy.

![Diagram](image)

**Figure 1:** Impartial decision model (IDM) for multi-criterion financial decision analysis
Hierarchy construction
As human-being is best to deal with complexity in a hierarchy (Herbert 1960); a two-dimension hierarchy about a decision problem is constructed first. Intangible information is categorized as a different criterion that is further classified as external, interactive and internal types. Profit conditions are assessed after establishing the overall benchmarked cash flow. Intangible criteria are then defined in every analysis such that a database of them is established progressively by a feedback mechanism.

Two-dimension entropy-based decision vector
Many physical quantities have the vector property of direction and magnitude. Velocity, weight and force are examples of vector. Decision weights $w_D$ are also vectors. Properties and attributes were distinguishable and interchangeable such that they were termed as criteria which were primitive (Saaty 1986). Vectors such as criteria, sub-criteria and alternatives in a decision share the weights in a space with directions relative to each other. Entropy has been useful in quantifying information transmission to deal with uncertainty in decision-making. In human brain, the signals transmission is performed by neural firing (Saaty 1996). There is imprecision about the possibility of a specific outcome such as the opinion of respondents on the significance of criteria in the construction industry. Elimination of uncertainty is a way to provide information. The entropy value actually represents the probability distribution of the significance of the criteria being studied. Shannon (1948) was the first one to build a measure of uncertainty for a probability distribution $p = (p_1, p_2, \ldots, p_n)$ and found the key measure as:

$$H(p_1, p_2, \ldots, p_n) = - \phi \sum_{i=1}^{k} p_i \ln(p_i)$$

where $H =$ entropy; $\phi_k = 1/\ln(k)$ is a positive constant which guarantees $0 \leq H(p_1, p_2, \ldots, p_n) \leq 1$; $i =$ a constant from 1 to $k$; and $k =$ number of scales. The expected value can be included to minimize bias using a decision matrix. In extreme cases, the use of the entropy method is possible in a multi-objective environment for non-competitive criteria. The newly-defined equation:

$$PR_m = \frac{\text{Expected value of the five scales of significance level, } E_m}{e^{\text{Entropy of those criteria, } H_m}}$$

$$= \frac{\left( \sum_{i=1}^{k} S_i \times p_{mi} \right)}{e^{\sum_{i=1}^{k} p_{mi} \ln(S_i)}}$$

where $PR_m =$ priority rating of $V_m$; $H_m =$ entropy of $V_m$; $E_m =$ expected value of $V_m$; $i =$ a constant from 1 to $k$; $k =$ number of scales; $S_i =$ scale of a degree of significance; and $p_{mi} =$ probability of a scale. The final weight of $V_m$ is computed by normalizing the priority ratings.

Let $\zeta$ be a finite set of $j$ elements called alternatives with decision weight of $\Psi_j$. Let $\delta$ be a set of $k$ criteria with decision weight $\Psi_k$ with respect to which elements in $\zeta$ are evaluated. In between the sets of $\zeta$ and $\delta$, there is a set of $i$ sub-criteria annotated $\psi$ with decision weight $\Psi_{i\delta}$. When a decision is made at time $t$, an entropy-
Impartial decision models

A two-dimension plane is formulated as the inclusion of 4 principal vectors $R_1^2, R_2^2, R_3^2, R_4^2$: 1) the relative weight between sub-criterion $SC_i$; 2) the relative weight between alternative $A_j$; 3) the relative weight between criterion $C_k$; and 4) the weights of criteria for each sub-criterion arising on each alternative $C_k \rightarrow SC_i \rightarrow A_j$. The entropy-based method is best adopted to evaluate the weights of criteria in the constructed hierarchy. A two-dimension plane is formulated in Step 2 as the inclusion of the four principal vectors as shown in Fig. 2. As the construction task is activity-based, the first principal vector is free of uncertainty and thus is used for consistency check and uncertainty quantification. Risk adjusted discounting rate (RADR) of each vector is incorporated with the weights derived by the two-dimension plane to obtain the final decision weights.

**Figure 2**: Two-dimension entropy-based decision vector

Uncertainties in each level of the hierarchy are quantified by an adaptive genetic algorithm (AGA) such that an impartial decision can be provided for the best, median and worst scenarios. Basically, a GA undergoes the search process in four stages: initialization, selection, crossover, and mutation (Davis 1991). Since GA has an inherently dynamic and adaptive nature, the change of the strategy parameter values during the optimization process is necessary by using: a) a rule; b) a feedback mechanism; and c) a self-adaptive mechanism (Gen and Cheng 2000) to reach the evolutionary principle. In this study, the second method is used to determine the direction of the change in the strategy parameter. The GA optimization software, Evolver™ (Palisade 1998), is used as it provided a steady state (Cheung et al. 2002). In the presence of an impartial environment, an effective adaptive phase is included in the algorithm. As the construction task is activity-based, the first principal vector is free of uncertainty and thus is used for consistency check for sub-criteria in one alternative by equation (3) where $i = 1,2,3,\ldots N$; $j = 1,2,3,\ldots O$; and $k = 1,2,3,\ldots M$.

$$\text{Inconsistency} = \left( \sum_{k=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{O} \frac{R_i^k}{ \left( \sum_{k=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{O} R_i^k \right) \times 100 \%} \right) 	imes 100 \% \times \frac{1}{O \times N}$$

The third and fourth principal vectors are optimized to quantify the abounded uncertainties. First, assume all uncertainties are distributed throughout the level of criteria weight and there exists no bias between the sub-criterion, then $R_1^2 = SC_i = 1/i$. 

**3rd principal vector:**

**1st principal vector:**

**Resultant vector:**

**2nd principal vector:**

**4th principal vector:**

**Criterion**

**Sub-criterion**

**Alternative**
The quantification of criterion weight is optimized by the following objective function in equation (4):

$$\tilde{M}ax \quad f_1(X) = \sum_{k=1}^{M} R_j^2 = \sum_{k=1}^{M} C_k$$

where $f_1(X) =$ sum of criteria decision weight; $R_j^3 =$ the 3rd principal vector; $C_k =$ the $k^{th}$ criterion decision weight $\in \delta$; $M =$ maximum of $k$; $N =$ maximum of $i$; and $O =$ maximum of $j$. Then, assume all uncertainties are distributed throughout the level of relative weight of criterion to sub-criterion of alternative. The quantification of relative weight is optimized by the objective function in equation (5) with 10% range allowance.

$$\tilde{M}ax \quad f_2(X) = \sum_{l=1}^{M \times N \times O} R_j^4 = \sum_{l=1}^{M \times N \times O} C_{k \rightarrow SC_{i \rightarrow A_j}} = \sum_{l=1}^{M \times N \times O} RW_l$$

where $f_2(X) =$ sum of relative decision weight of criterion to sub-criterion of alternatives; $R_j^4 =$ the 4th principal vector; $C_{k \rightarrow SC_{i \rightarrow A_j}} =$ the $l^{th}$ relative decision weight of $k^{th}$ criterion to $l^{th}$ sub-criterion arising on $j^{th}$ alternative; and $l =$ the $l^{th}$ relative weight evaluation. An impartial resultant vector and hence an impartial second principal vector will result such that the final rating of alternative $j$ can be computed readily by equation (6).

$$\text{Rating of alternative } j = \left( \sum_{k=1}^{M} \sum_{i=1}^{N} R_j^5 \right) \times A_j$$

### Three-dimension risk adjusted discount rate (RADR)

Prior to the application of the fuzzy reasoning technique, linguistic subsets have to be assigned to the selected risk criteria. The popular functions used for justification are either the $S$ function or the $\pi$ function that are, in fact, combinations of two $S$ functions (Zadeh 1975). After the procedure of justification, the data is passed to the inference engine where a rule base is established. The relative weights of the $k^{th}$ criterion for the $i^{th}$ sub-criterion in the $j^{th}$ alternative are input and the base rules from January to December are selected. The membership functions are then computed. When a case (x) is evaluated, it is put into each rule in turn by the popular max-min operation, that is, $\mu(a \text{ is } C_a) = \min_{k=1}^{N} \mu(a \text{ is } C_a)$. For the $P^{th}$ rule, $\mu_{[a \text{ is } C_a]}(x) = \min_{k=1}^{N} \mu_{[a \text{ is } C_a]}(x)$ and the final result is given by the centre-of-gravity method, that is, the sum of scalar rules is divided by the sum of rules. Normalization of the criteria values is essential to the maximum operation (Hwang and Yoon 1981). Before normalization, the overall weights on profit and risk were, in turn, composed of the weights of each alternative, of profit and risk in each alternative and of each sub-criterion in each alternative. In addition, the weights on the sub-criteria of each alternative were actually calculated by normalizing the weighted membership functions. Finally, RADR of each vector $R_j^3$ with decision weight $\overrightarrow{R_j}$ is acted as a time frame $dt$ per annum by equation (7) and incorporated with the weights derived by the two-dimension plane to obtain final three-dimension realistic decision weights.

$$R_j^3 = \int_{t=0}^{12} \overrightarrow{R_j} \, dt$$
For example, RADR of the $4^{th}$ principal vector is given by,

$$R^3 \left(R^1, R^2, \ldots, R^n\right) = \int_{t=1}^{12} \left(\overline{OP}_t - \overline{OR}_t\right) / 12 \, dt$$

where $t$ is a month from January to December; $\overline{OP}_t$ = overall weight of profit relative to the risk for the $i^{th}$ sub-criterion in the $j^{th}$ alternative from January to December; and $\overline{OR}_t$ = overall weight of risk relative to the profit for the $i^{th}$ sub-criterion in the $j^{th}$ alternative from January to December.

**CASE STUDY**

**Problem background**

A medium-size building contracting firm that had four recently obtained projects was examined to evaluate the IDM. Table 1 shows the details of the four projects. The construction activities of the projects were broken down into five operations including substructure, superstructure, wet and dry trades, building services and external works. A 27-month period was used as the examination period for the four projects. Risk criteria were first identified as external and interactive type including heavy rainfall, shortages of labour, time constraints and delays in payment. Payment terms were based on the measurement of work done, simply with 2-month delay allowing time for issuance of payment certificate from client to the contractor after measurement and for submission of the certificate for payment from the contractor.

**Table 1: Details of the projects**

<table>
<thead>
<tr>
<th>Project type</th>
<th>Public housing</th>
<th>Public housing</th>
<th>Public housing</th>
<th>Civil works</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract sum, HK$ in millions</td>
<td>6.85</td>
<td>18.40</td>
<td>10.19</td>
<td>14.69</td>
</tr>
<tr>
<td>Contract period, month(s)</td>
<td>9</td>
<td>21</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>Start, month</td>
<td>10$^{th}$</td>
<td>3$^{rd}$</td>
<td>5$^{th}$</td>
<td>14$^{th}$</td>
</tr>
<tr>
<td>Profit margin, %</td>
<td>17.5</td>
<td>20.0</td>
<td>18.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Certificate and payment period</td>
<td>21 days and another 21 days</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retention, HK$ in millions</td>
<td>1.5% or max.</td>
<td>1.5% or max.</td>
<td>1.5% or max.</td>
<td>1.5% or max.</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>2.2</td>
<td>1.58</td>
<td>max. 0.2</td>
</tr>
</tbody>
</table>

A survey was conducted between December 2001 and January 2002 to investigate the opinions of experts on various aspects of significance stated below. Totally, there were 26 respondents. Among them, 4 were government officials, 4 were bankers, 5 were contractors, 9 were developers and 4 were engineering consultants. A 5-point scale (1=insignificant; 2=slightly significant; 3=moderately significant; 4=highly significant; and 5=extremely significant) was used in the questionnaire to investigate the following four aspects: (a) the degree of relative significance of projects 1 to 4 (A1 to A4); (b) the degree of relative significance of having heavy rainfall (C1), shortages of labour (C2), time constraints (C3) and delays in payment (C4); (c) the degree of significance of various types of risk (C1 to C4) for each activity including substructure (SC1), superstructure (SC2), wet and dry trades (SC3), building services (SC4) and external works (SC5) arising for various projects; and (d) The degree of significance of profit and various types of risk (C1 to C4) arising for various projects.
RESULTS AND ANALYSIS

Through the IDM shown in Fig. 1, the financial risks of multiple-projects being undertaken by a medium-size construction firm in Hong Kong were assessed. The hierarchy of risk is constructed as shown in Fig. 3.

Project assessment without uncertainties

After quantifying the uncertainties by the AGA, twelve decision vectors in three principal vectors were found with different amount of uncertainties. The amount of uncertainties ranged from -44.16% in project 4 (A4) to 29.19% in project 1 (A1) in the second principal vector. With the lowest contract sum (17.5%), the smallest contract sum (only 6.85 millions) and the shortest contract period (9 months), the potential development of A1 was solely under-estimated by one-third while the other projects 2-4 (A2-4) were over-estimated. With the highest profit margin (25%), the second largest contract sum (14.69 millions) but relatively short contract period (13 months), A4 became the most uncertain project. Project 2 (A2) was the most certain project because its longest contract period (21 months) helped in spreading the risk due to its largest contract sum and the second highest profit margin. The amount of uncertainties ranged from -30.64% in shortages of labour (C2) to 12.31% in having heavy rainfall (C1) in the third principal vector. C1 was solely under-estimated as the impact of weather on construction has been overlooked comparatively. However, C1 was the most certain risk criterion among the others as the weather summary and forecast has been well performed by the Royal Observatory of Hong Kong. C2 was the most uncertain risk criterion as the labour market has been fluctuated in these few years. The amount of uncertainties ranged from -1.41% in the average relative weight of all risk criteria to all activities of project 3 (C to SC of A3) and -0.93% in the average relative weight of all risk criteria to all activities of project 1 (C to SC of A1) in the fourth principal vector. All average relative weights were over-estimated. C to SC of A3 and that of A4 were the two most uncertain relative weights as there were a lot of uncertainties abounded in A3 and A4 already.

![Figure 3: Constructed hierarchy in the case study](image-url)
By equation (6), the results of the project assessment were shown in Fig. 4. The range of ratings in descending order was project 4 (A4) (1.17 to 3.90), project 3 (A3) (1.14 to 3.47), project 1 (A1) (0.70 to 1.64) and project 2 (A2) (1.28 to 1.76). The upper bound unbiased rating of A1 was 2.3 times greater than its lower bound, the decision-makers were quite uncertain in the inherent risk. Together with its lowest profit margin (17.5%), it should be discarded. However, the potential development of A1 has been under-estimated by one-third. The cash outflow of A1 could be compensated by its cash inflow. A1 could proceed to forecast its risk adjusted discount cash flow to make the final decision. The upper bound unbiased rating of A2 was only 1.4 times greater than its lower bound. The decision-makers were quite certain in the inherent risk of A2. Together with its attractive profit margin (20%), it was selected. The upper bound unbiased rating of A3 was 3 times greater than its lower bound, the decision-makers were very uncertain about the risk of it this time. Together with its unattractive profit margin (only 18%), it was discarded eventually. A4 was the most uncertain project. At the same time, the upper bound unbiased rating of A4 was also 3.3 times greater than its lower bound. The decision-makers were very uncertain. However, due to its highest and thus attractive profit margin (25%), it was selected to proceed to forecast its risk adjusted discount cash flow before making the final decision.

![Figure 4: Project assessment in terms of ratings](image)

**Risk adjusted discount cash flow on projects**

By equation (7), RADRs of different project were computed. The RADR of A1 was -17.4%, A2 was -3.6%, A3 was -30.4% and A4 was 16.6% per annum. The cash flow profile of A3 was discounted by one-third during the assessment period of 27-working-month because the risk had a very great impact on it based on the impartial judgments of decision-makers and the experience of the contractor. This further supported the discard of A3. The data from projects 1, 2 and 4 were combined to plot an S-curve as shown in Fig. 5. Basically, a compromised solution of the construction schedule was achieved. The figure showed that the original forecasted or risk discounted optimal progress curves approached the late schedules. This meant that the contractor had inadequate resources to take on a new project during the construction period. The contractor was very likely risk-averse throughout the year. This was obviously due to the adverse economic situation after the financial crisis in 1997. The introduction of RADR in the overall cash flow model had no effect on the completion
month of the multiple projects. However, the risk adjusted discount overall cash flow was different from the original forecasted. In the early stages, only project 2 was operating. Since project 2 had small discounting effect, there were little discrepancies in the overall cash flow with and without discounting. From working months 10 to 15, the risk adjusted discount curves shifted down a little. This was due to the large discounting effect of project 1 that started operating. After working month 14, the introduction of project 4 became dominant. As project 4 had a positive discounting effect, it caused the whole risk adjusted discount profile to move upwards in order to reach the original early schedule. This effect remained and propelled the profile beyond the original early schedule in the working month 18. The risk adjusted discount overall profit at the end was 0.97 millions more than that of the original forecasted profit (20.84 millions). The cash flow profiles of projects were predicted accurately with the most adverse financial condition shown to the contractor. Thus the contractor can make decision to accept or reject projects and adopt suitable management strategies to mitigate the risk level to an acceptance level so that the corporate cash flow is improved, reducing the chance of insolvency or bankruptcy. However, the appropriate number of new projects depends on the contractor’s financial position, the resources available, and the business environment.

![Figure 5: Overall cash flow with and without risk adjusted discount](image)

**CONCLUSIONS**

An entropy-based IDM in three dimensional vectors has been presented for the solution of multi-criterion financial decision analysis in the construction industry. The financial risks of multiple projects being undertaken by a medium-size construction firm in Hong Kong were assessed to evaluate the model. The results indicated that uncertainties in each vector could be quantified to provide an upper, mean and lower bound financial risks on projects with inconsistencies 0.009, 0.032 and 0.036 for all the sub-criteria in one alternative respectively. RADRs of different projects provide a more accurate, objective and realistic decision on financial risk analysis to the decision-maker to further select and reject a project even if it has a high profit margin,
and control the projects by discounting the cash flow in terms of derived risk rate. Finally, a benchmarked overall cash flow was achieved in the selected projects. The risk adjusted discount overall cash flow was 0.97 millions more than the original forecasted (20.84 millions). The cash flow profiles of projects were predicted accurately with the most adverse financial condition shown to the contractor.

REFERENCES


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