A FUZZY REASONING DECISION MAKING APPROACH BASED MULTI-EXPERT JUDGEMENT FOR CONSTRUCTION PROJECT RISK ANALYSIS

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With the advance of technology, construction projects have become increasingly complex and dynamic. Risk assessment, therefore, is normally a process of decision making conducted by multiple experts in a vague and fuzzy environment. It should be noted that background and contribution of the experts to the subject under consideration would differ from one another. However, preferences of experts do not have to be in conflict. The decision to be made by a group is highly dependent on the way of interaction and the type of aggregation to individuals. Therefore, risk analysts are required to capture the underlying subjective preferences to reach a decision value upon a reliable analysis. In order to facilitate the handling of uncertainty and subjectivity associated with projects and expert judgements, a multi-expert decision making methodology based on fuzzy set theory is proposed. The methodology utilizes a fuzzy aggregation system in which an appropriate control action can be determined by the acquisition and examination of individual expert judgements. This paper describes the principal issues of multi-expert judgement based on fuzzy reasoning decision making approaches in construction project risk analysis. A risk analysis model based on fuzzy set theory is presented to facilitate the decision making involving multiple experts under a vague and subjective circumstance. An illustrative example is also presented to demonstrate the proposed risk analysis model.

Keywords: Fuzzy set theory, multi-expert judgement, decision making, risk assessment.

INTRODUCTION

Construction project risk assessment is normally a process of decision making conducted by multiple experts upon the available knowledge and information (An *et al* 2005; Zeng *et al* 2004). It is important that the involved experts have the right experience, skills and expertise and ideally the aspect of compatibility. Chapman (1998) declared that a balanced and effective group is normally formed by a particular combination of personality traits.

Accidences arise due to multiple root causes: incomplete design, poor communication, inadequate supervision, defective materials and improper construction techniques (HSE 2003a). In terms of a particular hazard, however, risk magnitude highly depends on many factors, such as the type and scope of the project, the qualification and experience of the contractor, adopted construction programme, site conditions, safety behaviour and safety performance of the construction workforce. In practice, statistic

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data obtained from the past only reflect a general image of the risk and the analysis of various factors involves substantial uncertainty and subjectivity. The nature of construction projects has undermined the applicability of many traditional risk assessment techniques which are widely used in the UK.

This paper presents a new risk analysis model based on fuzzy set theory to facilitate the decision making involving multiple experts in a vague and fuzzy environment. A case study has been used to illustrate the application of the methodology.

THE PROPOSED MULTI-EXPERT DECISION MAKING MODEL

In a multi-expert decision making system, an expert is not against problems and making the decision alone. He/She interacts with other experts to create a cooperated solution in which the wisdom of participated experts is assembled and the best decision is presented. Experts in a risk assessment group are viewed as a single decision maker and decision can only be made by the group not by the individual experts. Therefore, individual preferences do not have to be in conflict and the primary concern of decision making is to aggregate individual preferences made by each expert to a group preference.

In order to facilitate the handling of uncertainty and subjectivity associated with construction projects and expert judgements, a multi-expert decision making methodology based on fuzzy set theory is proposed and shown in Figure 1. The new fuzzy model algorithm is developed in four main stages and a general description of the model is given in the following sections.

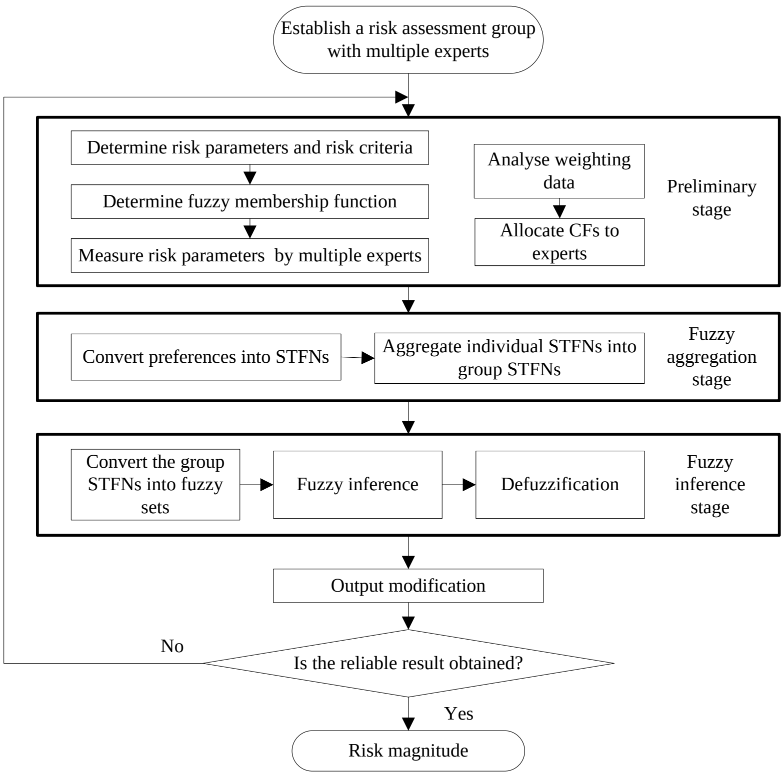

Figure 1. The fuzzy-based multi-expert decision making model

1. The preliminary stage

Risk assessment starts upon the completion of risk identification, in which all potential hazards have been identified and classified into different categories. In the preliminary stage, a risk assessment group is form by experts with essential experience and expertise in connection with the risk.

1.1 Contribution factors (CFs)

In order to distinguish the competence among experts, contribution factor (CF) is introduced to the fuzzy model. CF is determined by the analysis of experts' skills, experience and expertise to the risk under consideration. Assume an expert E_i, $i = 1,2,...,n$, is assigned a contribution factor c_i, where $c_i \in [0,1]$, $i = 1,2,...,n$ and $c_1 + c_2 + ... + c_n = 1$.

1.2 Risk parameters and risk criteria

Some risk parameters are used widely in judging risk magnitude, such as risk likelihood, risk severity, risk timing, detectability and consequence possibility. However, risk likelihood and risk severity are frequently used as two fundamental parameters for risk assessment.

Risk criteria are standards which define the scope and the context of risk magnitude and risk parameters. They may be different according to the change of time, project, stakeholder and internal and external circumstances. The risk parameters and risk criteria should be properly defined before the assessment taking place.

1.3 Fuzzy membership functions

Risk analysts are required to determine the fuzzy membership functions of risk magnitude and the chosen risk parameters. Fuzzy membership function usually stems from experimental data, perception of the linguistic terms and the simulation of reality. Besides, it should characterize the defined linguistic variables and be accommodated to the environment under consideration. In literature, several geometric mapping functions have been widely adopted, such as triangular-shaped function, trapezoidal-shaped function and S-shaped function.

2. The fuzzy aggregation stage

2.1 The standardised trapezoidal fuzzy number (STFN)

Experts within the risk assessment group are required to provide their evaluations to the chosen risk parameters under the defined criteria and domain. According to the nature of the risk and the availability of information and knowledge, each expert's preference can be a precise numerical value, a range of numerical values, a linguistic term, and a fuzzy number *etc*.

Normally, if adequate information is obtained and the risk parameter is quantitative measurable, an expert probably provides a precise numerical value or a probable range of numerical values as his/her preference. However, in most cases, experts find that it is hard to give numerical values due the involvement of uncertainty or the risk parameter is quantitative immeasurable, then a linguistic term or a fuzzy number can be used in the proposed fuzzy model. For example, "the risk likelihood is medium", "the risk severity is about high with 70% confidence" and "the risk likelihood is around 5 to 8 and most likely to be 7 in the universe of [0,10]".

All these numerical or fuzzy data can be converted to standardised trapezoidal fuzzy numbers (STFNs) which are relatively easy and intuitive to use by decision makers. STFNs act as a medium for representing experts' preferences and group preference. The STFN of parameter j measured by expert E_i can be defined by a quadruplet

$RP_{ij}* = (a_{ij}{}^{l}, a_{ij}{}^{m}, a_{ij}{}^{n}, a_{ij}{}^{u})$ in the universe of discourse $U = [0, u]$. Its membership function, as shown in Figure 2, is

$$\mu_{RP_{ij}*}(x) = \begin{cases} (x - a_{ij}{}^{l})/(a_{ij}{}^{m} - a_{ij}{}^{l}) & for \; a_{ij}{}^{l} \le x \le a_{ij}{}^{m}, \\ 1 & for \; a_{ij}{}^{m} \le x \le a_{ij}{}^{n}, \\ (a_{ij}{}^{u} - x)/(a_{ij}{}^{u} - a_{ij}{}^{n}) & for \; a_{ij}{}^{n} \le x \le a_{ij}{}^{u}, \\ 0 & for \; otherwise. \end{cases} \quad (1)$$

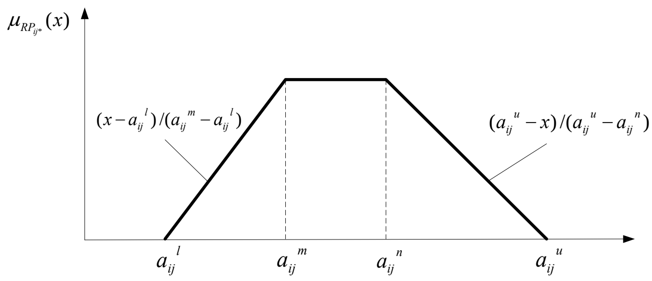

Figure 2. Membership function of the STFN

where $0 \le a_{ij}{}^{l} \le a_{ij}{}^{m} \le a_{ij}{}^{n} \le a_{ij}{}^{u} \le u$. In a STFN, membership function indicates the degree of preference made by experts, namely, $a_{ij}{}^{l}$ stands for the lower bond of the preference, $a_{ij}{}^{u}$ stands for the upper bond of the preference, and values within the interval [$a_{ij}{}^{m}, a_{ij}{}^{n}$] have the highest preference with a membership function of 1. A numerical value, a range of numerical values and a triangular fuzzy number can be viewed as simplified STFNs: when $a_{ij}{}^{l} = a_{ij}{}^{m} = a_{ij}{}^{n} = a_{ij}{}^{u}$, a STFN is reduced to a numerical number; when $a_{ij}{}^{l} = a_{ij}{}^{m}$ and $a_{ij}{}^{n} = a_{ij}{}^{u}$, a STFN is reduced to a range of numerical values; when $a_{ij}{}^{m} = a_{ij}{}^{n}$ a STFN is reduced to a triangular fuzzy number.

In this case, experts build a series of STFNs corresponding to the chosen risk parameters. Every STFN represents a preference which is provided by an expert based on the available information and his/her subjective judgement.

2.2 Aggregation of individual preferences

The aim of aggregation is to combine or reconcile individual preferences to a group preference so that a control action is determined. The aggregation can be obtained by applying the fuzzy weighted trapezoidal averaging operator. The process is defined as:

$$RP_j *_{agg} = c_1 \otimes RP_{1j} \oplus c_2 \otimes RP_{2j} \ldots c_n \otimes RP_{nj} \quad (2)$$

where $RP_j *_{agg}$ is the fuzzy aggregated result, i.e. the group preference of parameter RP_j; $RP_{1j}, RP_{2j}, \ldots, RP_{nj}$ are the STFNs of parameter j measured by expert $E_1, E_2, \ldots, E_n$, respectively; $\otimes$ and $\oplus$ stand for the fuzzy multiplication operator and the fuzzy addition operator, respectively. c_1, $c_2, \ldots$, and c_n are CFs allocated to experts, where c_1 assigned to E_1, c_2 assigned to E_2 and so on. $c_1 + c_2 + \ldots + c_n = 1$.

Through the fuzzy aggregation stage, different forms of individual preferences provided by experts have been converted into STFNs and aggregated into group preferences with each parameter assigned a distinct STFN which adequately represents the evaluation made by multiple experts.

3. The fuzzy inference stage

In the fuzzy inference stage, risk analysts input the STFNs into the fuzzy inference system, decide the extent to which rules relevant to the current situation, then calculate the fuzzy output of risk magnitude *RM** by using the information stored in the rule base and defuzzify the fuzzy result into a matching numerical value which adequately represents *RM**.

3.1 Convert the STFNs into matching fuzzy sets for fuzzy inference

The aggregated STFNs are not always can be directly used in a fuzzy inference system, for example, the rules stored in the rule base are constructed by linguistic terms and STFNs are presented in numerical values. In this case, risk analysts are required to convert the STFNs into matching fuzzy sets which are favourable to the fuzzy inference system. There are several methods in conducting this conversion available in literature. One easy way is taking intersections between the STFN and the membership function of the corresponding risk parameter. An example of this method is shown in the case study.

3.2 Fuzzy inference

A risk is measured by the composition of certain risk parameters, such as risk likelihood, risk severity and risk timing (An *et al* 2005; Zeng *et al* 2004; HSE 2003b). The fuzzy inference system provides an effective method to deal with imprecise and vague information associated with construction risks (Sousa and Kaymak 2002; Pillay and Wang 2003). In this system, expert judgements and heuristic knowledge are captured and stored in the knowledge base. Relations between risk parameters and risk magnitude are interpreted in a form of *if-then* rules.

Assume that the inference system has *m* inputs $RP_1*, RP_2*, \ldots, RP_m*$ and one output *RM**. The *if-then* rules can be written as:

$$R^k: \text{ If } RP_1* \text{ is } A_1^k \text{ and } RP_2* \text{ is } A_2^k \text{ and } \ldots \text{and } RP_m* \text{ is } A_m^k \text{ then } RM* \text{ is } B^k \quad (3)$$

where A_1^k, $A_2^k, \ldots, A_m^k$, and B^k denote membership functions of risk parameter $RP_1*, RP_2*, \ldots, RP_m*$, and risk magnitude *RM**, respectively; R^k, k=1,2,…,K is the kth rule stored in the rule base. The fuzzy inference system thus generates a mapping between antecedent - risk parameter $RP_1*, RP_2*, \ldots, RP_m*$ and consequence - risk magnitude *RM**.

Implication operation is applied between antecedent and consequent. Under Mamdani's minimum operator, a fuzzy rule for risk inference can be represented by the membership function as follows:

$$\mu_{R^k}(\chi, y) = \mu_{RP_1}{}^k(x_1) \wedge \mu_{RP_2}{}^k(x_2) \wedge \ldots \wedge \mu_{RP_m}{}^k(x_m) \wedge \mu_{RM}{}^k(y), k = 1,2,\ldots,K \quad (4)$$

where $x_1 \in X_1, x_2 \in X_2, \ldots, x_m \in X_m, \chi \in X_1 \times X_2 \times \ldots \times X_m$ and $y \in U$. $X_1, X_2, \ldots, X_m$, U denote the universe of $RP_1*, RP_2*, \ldots, RP_m*$ and *RM**, respectively. The total fuzzy relation R can be found by aggregating each fuzzy relation. For example, since a Mamdani's minimum operator is used in Eq. (4) for interpreting R^k, now the maximum operator taking the union of individual rules can be used to obtain the total relation given by the membership function as follows:

$$\mu R(x, RM^*) = \vee_{k=1}^{K} R^k(x, RM^*) \tag{5}$$

There are two principal methods of inference in fuzzy systems: the *min-max* method and the fuzzy additive method (Cox 1999). The fuzzy output *RM** is found by composing the fuzzy input *RP** with the total relation that is described by the fuzzy rules. Given fuzzy input *RP**, the fuzzy output *RM** is

$$RM^* = RP^* \circ R(x, y) \tag{6}$$

where symbol "$\circ$" denotes the compositional operation of fuzzy sets.

3.3 Defuzzification

A numerical value of risk magnitude is required in many cases. Since the output of the fuzzy inference system is a fuzzy set, defuzzification is used for converting the fuzzy result into a matching numerical value that can adequately represent *RM**. The well developed defuzzification techniques include centre of gravity (COG), centre-average and mean of maximum.

Centre-average is a proven simple and plausible approach frequently used for defuzzification. Assume the fuzzy output obtained from the fuzzy inference system is $RM^* = \{(y, \mu_{RM}(y) | \, y \in U, \mu_{RM} \in [0,1]\}$ with p fuzzy term sets, the matching crisp value *RM* can be obtained as follows:

$$RM = \left(\sum_{i=1}^{p} Y_i \mu_{RM}(y_i) \right) / \left(\sum_{i=1}^{p} \mu_{RM}(y_i) \right) \tag{7}$$

where $i = 1,2,..., p$. Y_i denotes the centre of the fuzzy term set *i* of RM^*, and $\mu_{RM}(y_i)$ denotes the membership function of the fuzzy term set *i* of RM^*.

4. The output modification stage

One should notes that there are a number of techniques available for fuzzy aggregation, fuzzy inference and defuzzification. In terms of a particular case, change of operator sometimes results in a substantially different decision value. Moreover, output modification is also necessary in some situations for securing a reliable decision, for instance, the circumstances of risks have been changed, the output risk magnitude falls into a dilemma, such as with a belief of 50% for tolerable and with a belief of 50% for intolerable. In this case, Experts and risk analysts should gather more information and evidences related to the risk, review the risk assessment process and modify the risk parameter or/and risk criteria to reach a reliable decision.

Through the application of the proposed fuzzy-based risk assessment methodology, the defined risk can be carefully assessed and risk magnitude is determined. The final result provides the project management team with reliable data for risk respond decision making.

AN ILLUSTRATIVE EXAMPLE

A risk management project team is formed to manage risks arising in the demolition of a commercial building. Demolition is regarded as a dangerous and detrimental activity and the risk of fire is now required to assess.

Many causes can lead to a fire while demolition taking place, such as electrical faults, rubbish burning out of control, heaters of all kinds, misoperation of storage and use of flammable materials (King and Hudson 1985). However, in terms of a particular demolition, many factors can influence the risk magnitude of fire, such as the age and condition of the building, the details and location of public services, the adopted demolition method and equipment, the qualification and experience of demolition workers. Since there are no adequate practical data and information to support a traditional risk analysis, the proposed fuzzy-based risk assessment methodology is employed to facilitate the decision making upon the underlying uncertainties.

The application of the proposed methodology consists of four stages and can be described as follows.

1. The preliminary stage

Five experts with high qualification regarding this subject are selected to form a risk assessment group for undertaking the risk assessment by using the proposed fuzzy-based risk assessment methodology. According to the analysis of their individual backgrounds, a contribution factor (CF) is assigned to each expert as shown in Table 1.

Table 1 Assigned contribution factor of experts

Experts	Background	Contribution factor
E_1	Safety manager	0.24
E_2	Construction manager	0.23
E_3	Senior engineer	0.20
E_4	Site engineer with 20 years experience	0.18
E_5	Site engineer with 8 years experience	0.15

Risk likelihood (*RL*) and risk severity (*RS*) are chosen as two parameters for assessing the risk magnitude (*RM*) of fire while demolition of the commercial building. Five experts agree that five levels of linguistic variables are used for the expression of *RL* and *RS*: *very low* (*VL*), *low* (*L*), *medium* (*M*), *high* (*H*) and *very high* (*VH*). On the other hand, risk magnitude (*RM*) is expressed in three levels: *negligible* (*N*), *tolerable* (*T*) and *intolerable* (*I*). The risk assessment group agrees to describe the linguistic terms of *RL*, *RS* and *RM* as shown in Table 2, 3 and 4, respectively.

Table 2 Risk Likelihood (*RL*)

Description	General interpretation	Occurrence rate
Very low	Occurrence is unlikely	Below 10^{-9}
Low	Occasionally happen	$10^{-7}$to 10^{-9}
Medium	Likely to happen	10^{-5} to 10^{-7}
High	Frequently happen	10^{-3} to 10^{-5}
Vary high	Occurrence is almost inevitable	10^{-0} to 10^{-3}

Table 3 Risk Severity (*RS*)

Description	General interpretation
Very low	Fire damage is ignorable.
Low	Single minor injury, likely business loss within ￡10, 000.
Medium	Single major injury or multiple minor injuries, likely business loss ￡10, 001−100, 000.
High	Single fatality or multiple major injuries, likely business loss ￡100, 001−500, 000.
Very high	Multiple fatality or a number of major injuries, likely business loss greater than ￡500, 000.

Table 4 Risk Magnitude (*RM*)

Description	General interpretation
Negligible	The risk is low or insignificant and can be readily controlled.
Tolerable	The risk is medium and is tolerable in order to secure certain benefits. However, risk controls should be undertaken if it is reasonably practicable to do so.
Intolerable	The risk is unacceptable no matter what benefits associated with that risk. Proper action must be taken to eliminate or reduce the risk.

RL and *RS* are interpreted in triangular membership functions as shown in Figure 3 (adapted from Carr and Tah, 2001), where *very low* (*VL*) = (0,0,3), *low* (*L*) = (1,3,5), *medium* (*M*) = (3,5,7), *high* (*H*) = (5,7,9) and *very high* (*VH*) = (7,10,10). On the other hand, trapezoidal membership functions are introduced to *RM* as shown in Figure 4. The corresponding trapezoidal numbers are *negligible* (*N*) = (0,0,1,4), *tolerable* (*T*) = (1,4,6,9) and *intolerable* (*I*) = (6,9,10,10).

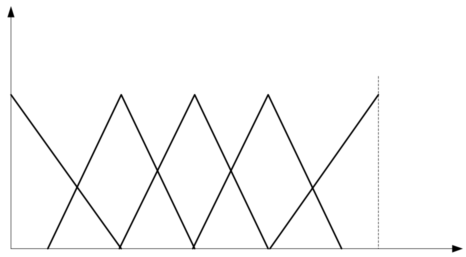

Figure 3. Fuzzy definition of *RL* and *RS*

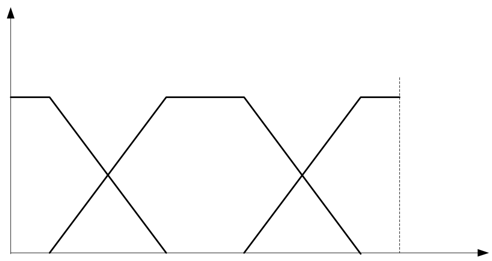

Figure 4. Fuzzy definition of *RM*

Five experts in the risk assessment group are now requested to give their evaluations to *RL* and *RS* under the defined criteria and score system, i.e. from 0 to 10, inclusive. Each expert can provide a precise numerical value, a probable range of numerical values, a linguistic term, or a fuzzy number subject to his/her knowledge and the available information in hand. Then these evaluations are converted into STFNs as defined in Eq. (1). In this case study, Expert E_1 and E_4 use linguistic terms, E_2 uses a precise numerical value, E_3 uses a range while E_5 provides a triangular fuzzy number containing three estimated values, namely the optimistic value, the most likely value and the pessimistic value as shown in Table 5.

Table 5 Evaluations and STFNs of *RL* and *RS*

Experts	Evaluation			
	RL		*RS*	
	Preference	Converted STFN	Preference	Converted STFN
E_1	Medium	(3,5,5,7)	High	(5,7,7,9)
E_2	4.5	(4.5,4.5,4.5,4.5)	7.0	(7.0,7.0,7.0,7.0)
E_3	(3,5)	(3,3,5,5)	(6,8)	(6,6,8,8)
E_4	About 4	(3,4,4,5)	About 7.5	(6.5,7.5,7.5,8.5)
E_5	(2,4,5)	(2,4,4,5)	(6,8,9)	(6,8,8,9)

2. The fuzzy aggregation stage

The fuzzy weighted trapezoidal averaging operator is chosen for the fuzzy multi-expert decision making. According to the Eq. (2), the aggregation process is shown in Table 6.

Consequently, the aggregated STFNs are:

*RL** = (3.195, 4.155, 4.555, 5.365), *RS** = (6.080, 7.040, 7.440, 8.250).

Table 6 Fuzzy aggregation

Risk parameter	Fuzzy aggregation
RL	a^l: $\mu(x)$ = 3×0.24+4.5×0.23+3×0.20+3×0.18+2×0.15 = 3.195
	a^m: $\mu(x)$ = 5×0.24+4.5×0.23+3×0.20+4×0.18+4×0.15 = 4.155
	a^n: $\mu(x)$ = 5×0.24+4.5×0.23+5×0.20+4×0.18+4×0.15 = 4.555
	a^u: $\mu(x)$ = 7×0.24+4.5×0.23+5×0.20+5×0.18+5×0.15 = 5.365
RS	a^l: $\mu(x)$ = 5×0.24+7.0×0.23+6×0.20+6.5×0.18+6×0.15 = 6.080
	a^m: $\mu(x)$ = 7×0.24+7.0×0.23+6×0.20+7.5×0.18+8×0.15 = 7.040
	a^n: $\mu(x)$ = 7×0.24+7.0×0.23+8×0.20+7.5×0.18+8×0.15 = 7.440
	A^u: $\mu(x)$ = 9×0.24+7.0×0.23+8×0.20+8.5×0.18+9×0.15 = 8.250

3. The fuzzy inference stage

Since the aggregated STFNs are not presented in the form of linguistic variables stored in the rule base, risk analysts therefore are required to convert them into matching fuzzy sets which are favourable to the fuzzy inference system. One easy way is taking the intersections between the STFN and the membership function of the corresponding parameter. For example, the aggregated STFN of risk likelihood is *RL** = (3.195, 4.155, 4.555, 5.365) as shown in Figure 5 (the thick segments), then the matching fuzzy set $\hat{RL}^*$ is obtained by taking the intersections between the STFN and the fuzzy term sets of *RL**, i.e.

$\hat{RL}^*$ = {(*low*, 0.610), (*medium*, 0.842), (*high*, 0.130)}

It is noted that (*medium*, 0.188) is included into (*medium*, 0.842). Likewise, one can obtain the matching fuzzy set of risk severity $\hat{RS}$ as

$\hat{RS}^*$ = { (*medium*, 0.311), (*high*, 0.990), (*very high*, 0.328)}

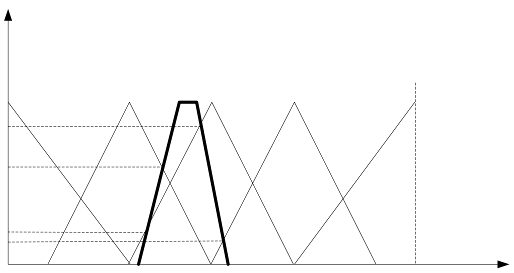

Figure 5. The matching fuzzy set $\hat{RL}^*$

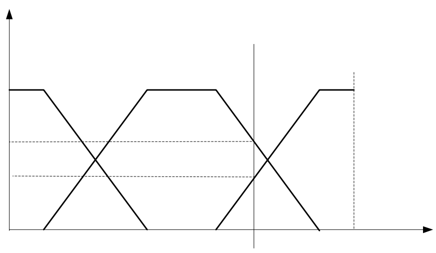

Figure 6. Defuzzification of *RM**

The risk assessment group produces 25 rules in the rule base as shown in Table 7, where *VL*, *L*, *M*, *H*, *VH*, *N*, *T* and *I* represent *very low*, *low*, *medium*, *high*, *very high*, *negligible*, *tolerable* and *intolerable*, respectively. These rules are interpreted as, for example: if *RL* is *very low* and *RS* is *very low*, then *RM* is *negligible*; if *RL* is *very low* and *RS* is *low*, then *RM* is *negligible*.

Table 7 Table of *if-then* rules

Experts		Risk likelihood (*RL*)				
		VL	*L*	*M*	*H*	*VH*
Risk severity (*RS*)	*VH*	*N*	*T*	*I*	*I*	*I*
	H	*N*	*T*	*T*	*I*	*I*
	M	*N*	*T*	*T*	*T*	*I*
	L	*N*	*N*	*T*	*T*	*T*
	VL	*N*	*N*	*N*	*T*	*T*

The *min-max* rule of implication is used in this case study. The principle of this method is using *minimum* operator in the fuzzy input region while taking the *maximum* operator to calculate these minimized fuzzy sets in the fuzzy output region. The fuzzy inference can be broken down into four steps as described below.

(1) Determining which rule is on in the rule base

From the mapping of inputs $RL^* \times RS^*$, the following 9 rules in Table 7 are fired:

R^1: If *RL* is *low* and *RS* is *medium*, then *RM* is *Tolerable*;
R^2: If *RL* is *low* and *RS* is *high*, then *RM* is *Tolerable*;
R^3: If *RL* is *low* and *RS* is *very high*, then *RM* is *Tolerable*;
R^4: If *RL* is *medium* and *RS* is *medium*, then *RM* is *Tolerable*;
R^5: If *RL* is *medium* and *RS* is *high*, then *RM* is *Tolerable*;
R^6: If *RL* is *medium* and *RS* is *very high*, then *RM* is *Intolerable*;
R^7: If *RL* is *high* and *RS* is *medium*, then *RM* is *Tolerable*;
R^8: If *RL* is *high* and *RS* is *high*, then *RM* is *Intolerable*;
R^9: If *RL* is *high* and *RS* is *very high*, then *RM* is *Intolerable*;

(2) Taking the minimum operator to calculate the strength of the fired rules

The process is shown as follows:

R^1: $\alpha_1 = \mu L\,(RL^*) \wedge \mu M\,(RS^*) = min\ (0.610, 0.311) = 0.311$
R^2: $\alpha_2 = \mu L\,(RL^*) \wedge \mu H\,(RS^*) = min\ (0.610, 0.990) = 0.610$
R^3: $\alpha_3 = \mu L\,(RL^*) \wedge \mu VH\,(RS^*) = min\ (0.610, 0.328) = 0.328$
R^4: $\alpha_4 = \mu M\,(RL^*) \wedge \mu M\,(RS^*) = min\ (0.842, 0.311) = 0.311$
R^5: $\alpha_5 = \mu M\,(RL^*) \wedge \mu H\,(RS^*) = min\ (0.842, 0.990) = 0.842$
R^6: $\alpha_6 = \mu M\,(RL^*) \wedge \mu VH\,(RS^*) = min\ (0.842, 0.328) = 0.328$
R^7: $\alpha_7 = \mu H\,(RL^*) \wedge \mu M\,(RS^*) = min\ (0.130, 0.311) = 0.130$
R^8: $\alpha_8 = \mu H\,(RL^*) \wedge \mu H\,(RS^*) = min\ (0.130, 0.990) = 0.130$
R^9: $\alpha_9 = \mu H\,(RL^*) \wedge \mu VH\,(RS^*) = min\ (0.130, 0.328) = 0.130$

(3) Determine the control outputs of fired rules

According to Eq. (4), the control outputs of fired rules can be obtained as follows:

R^1: $\alpha_1 \wedge \mu T\,(RM^*) = \min\ (0.311, \mu T\,(RM^*))$
R^2: $\alpha_2 \wedge \mu T\,(RM^*) = \min\ (0.610, \mu T\,(RM^*))$
R^3: $\alpha_3 \wedge \mu T\,(RM^*) = \min\ (0.328, \mu T\,(RM^*))$
R^4: $\alpha_4 \wedge \mu T\,(RM^*) = \min\ (0.311, \mu T\,(RM^*))$
R^5: $\alpha_5 \wedge \mu T\,(RM^*) = \min\ (0.842, \mu T\,(RM^*))$
R^6: $\alpha_6 \wedge \mu I\,(RM^*) = \min\ (0.328, \mu I\,(RM^*))$
R^7: $\alpha_7 \wedge \mu T\,(RM^*) = \min\ (0.130, \mu T\,(RM^*))$
R^8: $\alpha_8 \wedge \mu I\,(RM^*) = \min\ (0.130, \mu I\,(RM^*))$
R^9: $\alpha_9 \wedge \mu I\,(RM^*) = \min\ (0.130, \mu I\,(RM^*))$

It is noted that Rule R^1, R^2, R^3, R^4 and R^7 are included into Rule R^5; Rule R^8 and R^9 are included into Rule R^6.

(4) Taking the maximum operator to calculate the total relation

According to Eq. (5), the total relation is given by the membership function as follows:

$$\mu_{agg}(RM^*) = \max\{\min(0.842, \mu T(RM^*)), \min(0.328, \mu I(RM^*))\}$$

Defuzzification. This step is to convert the fuzzy output *RM** into a matching numerical value *RM* in describing the risk magnitude of fire. By using the centre-average defuzzification operator as shown in Eq. (7), *RM* is given as follows:

$$RM = \frac{6\times 0.842 + 10\times 0.328}{0.842 + 0.328} = 7.121$$

Consequently, the overall risk magnitude of fire while demolition of the commercial building is 7.121 under the defined scale system of *RM*, i.e. the risk is between *Tolerable* and *Intolerable* with a belief of 62.6% for *Tolerable* and 37.4% for *Intolerable* as shown in Figure 6.

4. The output modification stage

In reviewing the assessment process, the final result is found reliable so that no further output modification is needed. This result provides the risk management project team with a valuable data for risk response decision making.

CONCLUSIONS

Traditional risk assessment approaches and methodologies often do not capture the nature of construction projects particularly when they involve inherent subjectivity and uncertainty. On the other hand, the use of fuzzy set theory can provide a reliable tool to handle ill-defined problems and facilitate the decision making in a vague and fuzzy environment. The proposed risk assessment methodology has favourable flexibility for situations where fuzzy and/or crisp preferences are presented. It encodes knowledge directly in a way close to the natural thought of experts and the way of the decision making process, which makes the knowledge acquisition easier, more reliable and less ambiguities. Moreover, the methodology is particularly suitable for the risk analysis involving multiple experts and substantial uncertainties due to the existence of imprecise and incomplete information.

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