ANALYSIS OF DYNAMICAL IMPACTS OF INTEREST RATE ON EXPECTED HOUSING PRICE

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The first interest rate hike in China during the last decade aiming to cool the seemly overheated real estate market arouses debate on whether financial policy is indeed effective for housing price adjustment. Different real estate markets have different scenarios during a sudden change (shock) of interest rate. A cobweb model is built to analyze the after-shock oscillations. Consideration includes the heterogeneous expectations of agents, supply lag and deprecation rate. In particular, user cost demand model and stock-flow supply model are used. The results show that the dynamics of the expected housing price varies substantially with these factors. Financial policies should be chosen carefully in consistence with each unique real estate market, since some portfolio parameters can increase or suppress the price oscillations.

Keywords: cobweb model, expected housing price oscillations, heterogeneous expectations.

INTRODUCTION

The first interest rate hike during the last decade aims directly to monitor the heated real estate market in China. Whether it is effective to adjust the price oscillation is arguable. Previous researches show that a sudden interest rate change brings quite different results in different real estate markets (Pozdena 1990; Tse, etc. 1999; Gauger 2003; Meen 2002; Thomson 2004; Kahn 1990).

Modern economists tend to seek the cause of price oscillation in the real estate market by a unique shock. Some endogenous variables are chosen to study the oscillation, for examples, long supply lag time and property durability. With perfect foresight assumptions, such endogenous market cycles should not occur (Wheaton 1999; Poterba 1984). In traditional economic theory, the rational expectations hypothesis (REH) introduced by Muth (1961) dominates the paradigm of expectation formation. Relative to the REH, there is a rapidly growing literature on “bounded rationality” when agents use learning models (Hommes 2001) due to the fact that assets are not liquid due to their high values and their unique properties. Such phenomenon in the real estate market has been investigated (Case & Shiller, 1989).

The performance of the cobweb model is widely studied in economics (Buchanan, 1939; Stein, 1992). It started with the naive expectation model (Kaldor, 1934; Ezekiel, 1938) that the current expected price equals to the previous actual price. Nerlove (1958) first introduced the element of adaptive price expectations into the cobweb

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model. It assumed that the current expected price equals to the weighted average of the previous actual price and the previous expected price. An adaptive belief system is introduced by Brock and Hommes (1998, 2000) to model economic and financial markets. They assumed that traders choose their strategies according to an evolutionary fitness measure and that agents are rationally bounded.

This paper investigates the price dynamics due to an unanticipated interest rate change in the real estate market based on the cobweb model with heterogeneous expectations. In particular, user cost house demand and stock-flow supply model are used. Demand elasticity, supply elasticity, supply lag and depreciation rate are taken as varying parameters. The dynamics is described by a set of nonlinear difference equations taking the heterogeneous expectations of agents into consideration. Section 2 discusses the dynamical model with heterogeneous expectations. Equilibrium and local stability are analyzed. In section 3, parametric study of demand and supply elasticity, supply lag and depreciation rate of irrational expectations is given. This paper ends with some conclusions.

MODEL DEVELOPMENT

The cobweb model is one of the simplest economic models (Hommes 1994). It was introduced by Kaldor (1934) using the following three equations: (1)$D_t = d(p_t)$; (2)$S_t = s(\hat{p}_t)$; and (3)$D_t = S_t$. D_t, S_t, p_t, $\hat{p}_t$ are the demand of goods, supply of goods, actual price and expected price at time *t* respectively. The demand curve will be constructed in a specific functional form to simplify the analysis. The demanded quantity in the real estate market is assumed to depend proportionally on an exogenous economic variable and respond to price at time *t* with constant elasticity $-\beta_1$. Therefore the demand in a real estate market is characterized by

$$D_t = \alpha_1 \frac{1}{U_t} p_t^{-\beta_1} \qquad (1)$$

where, U_t is the user after-tax cost of housing ownership (Quigley 1999).

$$U_t = (1 - T_t) \times i_t \qquad (2)$$

where T is the normal income tax rate and i is the nominal interest rate. Assume the demand shock is a constant, $i_t = i,\ T_t = T$. The real estate market is often modelled within a stock-flow framework (Wheaton, 1999),

$$\frac{S_t}{S_{t-1}} = (1 - \delta) + \frac{C_{t-n}}{S_{t-1}} \qquad (3)$$

where δ is the depreciation rate and C_{t-n} is the new space delivery at time *t*. The supply lag is due to the longer production cycle in the real estate market so that the decision in acquiring new space is made at *n* periods before delivery time and the quantity depends on the estimate of price at delivery time *t* ($\hat{p}_t$).

$$\frac{C_{t-n}}{S_{t-1}} = \alpha_2 \hat{p}_t^{\beta_2} \qquad (4)$$

Equilibrium and local stability of dynamics model

According to $D_t = S_t$, a set of difference equations with time delay can be used to describe the expected price dynamics of the real estate market. The form of the difference equations depends on the heterogeneous expectations. The price oscillation cycle will be determined by the stability of the difference equations. Given that $\hat{p}^*$ is an equilibrium point of equation (4) satisfying $f(\hat{p}_t) = \hat{p}_t = \hat{p}^*$, such that

$$\frac{S_t}{S_{t-1}} = 1 \qquad \hat{p}^* = \left(\frac{\delta}{\alpha_2}\right)^{\frac{1}{\beta_2}} \tag{5}$$

An oscillation can be defined as the departure of the price from the steady state after the unexpected demand shock. Passing the steady state once only defines an over-shoot or under-shoot. Passing repeatedly the steady state with alternative processes of over-shoot and under-shoot defines as an oscillation cycle. Further, if the departure amplitude of the price from the steady state forms a decreasing trend approaching to the steady state gradually, it is convergent. Finally, if the departure amplitude from the steady state forms an increasing trend and departs away from the steady state gradually, it is defined as divergent or explosive.

Dynamics model with heterogeneous believes

Since the production of new space depends on the expected price at time *t* and the decision is made *n* periods before then, it is obvious that heterogeneous expectations of housing price affect the market dynamics.

Perfect foresight

. A perfect foresight is referred to the forecast when the real estate price equals to the present discounted value of the future rents. A rational expectation is usually equivalent to a perfect foresight. Poterba (1984) and Wheaton (1999) separately demonstrated that cycles cannot occur if the following two conditions can be satisfied: (1) the price at the delivery time of new space is based only on the future rents; and (2) estimates of prices at time *n* periods earlier than the delivery time are completely self-fulfilling. In this paper, the perfect foresight is based on the principle that the expected price is equal to the actual price $\hat{p}_t = p_t$ reflecting the meaning of correct forecast directly, i.e., the quantity demand changes proportionally to an unanticipated interest rate shock that agents can recognize and forecast correctly. The difference equation of the perfect foresight model is given by,

$$\left(\frac{\hat{p}_t}{\hat{p}_{t-1}}\right)^{-\beta_1} = (1-\delta) + \alpha_2 \hat{p}_t^{\beta_2} \tag{6}$$

We are going to prove by contradiction below that no oscillation exists in the perfect foresight model. If an oscillation exists and passes across the steady state, then there would be local maximum (minimum) points. Without loss of generality, assume $\hat{p}_t$ is the maximum point. In equilibrium, the RHS of equation (6) is equal to 1. The RHS of equation (6) will be greater than unity at $\hat{p}_t$. It is obvious that $\hat{p}_t < \hat{p}_{t-1}$. However, it contradicts the assumption that $\hat{p}_t$ is a local maximum. Similarly, we can prove that $\hat{p}_t$ is a local minimum. The above proof is consistent with the fact that in a perfect foresight the production of new space is determined by the expected price at time *t*. Too high an expected price results in overbuilding. But if it is indeed overbuilt, the actual price will fall eventually. It contradicts the assumption of perfect foresight that the expected price is equal to the actual price.

Naive expectation

The traditional cobweb model is based on the naive expectation assumption that the expected price at delivery time *t* equals to the actual price at decision time *t-n* given by $\hat{p}_t = p_{t-n}$. The price dynamics is governed by,

$$\left(\frac{\hat{p}_{t+n}}{\hat{p}_{t+n-1}}\right)^{-\beta_1} = (1-\delta) + \alpha_2 \hat{p}_t^{\beta_2} \tag{7}$$

Adaptive expectation

Adaptive expectations are introduced into the cobweb model to give the following equation:

$$\hat{p}_t = \omega p_{t-n} + (1-\omega)\hat{p}_{t-n}, \qquad 0 \le \omega \le 1 \tag{8}$$

It shows that the expected price at delivery time *t* is the weighted average of expected and actual prices at decision time *t-n* using the expectation weight factor ω. When $\omega = 1$, the model reduces to the traditional model of naive expectation. When $\omega = 0$, $\hat{p}_t = \hat{p}_{t-n}$, the expected price at delivery time *t* is equal to the expected price at decision time *t-n*. Equation (8) can also be written as:

$$\hat{p}_t = \hat{p}_{t-n} + \omega(p_{t-n} - \hat{p}_{t-n}) \qquad 0 \le \omega \le 1 \tag{9}$$

It shows that the expected price at delivery time *t* is a correction to the expected price at decision time *t-n*. The constant ω controls the degree of correction. The dynamical model of the adaptive expectation can be characterized as,

$$\left(\frac{\hat{p}_{t+n} - (1-\omega)\hat{p}_t}{\hat{p}_{t+n-1} - (1-\omega)\hat{p}_{t-1}}\right)^{-\beta_1} = (1-\delta) + \alpha_2 \hat{p}_t^{\beta_2} \tag{10}$$

Biased belief

Estate agents have different preferences to the risks and profits in the economic theory. They have unique views on the expected price in a biased expectation,

$$\hat{p}_t = p_{t-n} + h \tag{11}$$

where, h is the bias coefficient. If agents have a continuous optimistic expectation on the price, then $h > 0$. When they are continuously pessimistic, then $h < 0$. In particular, when $h = 0$, the model reduces to the traditional naive expectation model. The dynamical model of biased expectation can be described as,

$$\left(\frac{\hat{p}_{t+n} - h}{\hat{p}_{t+n-1} - h}\right)^{-\beta_1} = (1-\delta) + \alpha_2 \hat{p}_t^{\beta_2} \tag{12}$$

Trend following expectation

Trend follower phenomenon is one of the most famous topics of bounded rationality which has been investigated for a long time. It can be described by:

$$\hat{p}_t = p_{t-n} + g \times (p_{t-n} - p^*) \tag{13}$$

It implies the belief of agents that the new price always follows the previous price trend and the change rate is equal to g for $g > 0$.

With trend following expectation, agents believe that the increasing or decreasing trend at decision time *t-n* will be kept at delivery time *t*. The dynamical model can be characterized as,

$$\left(\frac{\hat{p}_{t+n} - g \times p^{*}}{\hat{p}_{t+n-1} - g \times p^{*}}\right)^{-\beta_1} = (1 - \delta) + \alpha_2 \times \hat{p}_t^{\beta_2} \qquad (14)$$

When some parameters in equations (12) or (14) are fixed, for example $h = 0$ or $g = 0$, the biased expectation model or trend following expectation model will converge to the naive expectation model. The following simulations show that such expectations based on backward-looking actual data are qualitatively similar but quantitatively different.

MODEL DYNAMICS WITH IRRATIONAL EXPECTATIONS

In this part, we investigate the dynamical behaviour of heterogeneous expectations in the real estate market by parametric study on the demand elasticity β_1, supply elasticity β_2, time lag n and depreciation rate δ. The other parameters are fixed at $p^{*} = \$400$ per square foot and i=5%. First, in order to compare the results of Wheaton (1999), make the same assumption that the demand and supply elasticities are both equal to 1, the depreciation rate is 0.05 and the supply lag is 5 years. Figure 1 shows the results of the heterogeneous irrational expectation models.

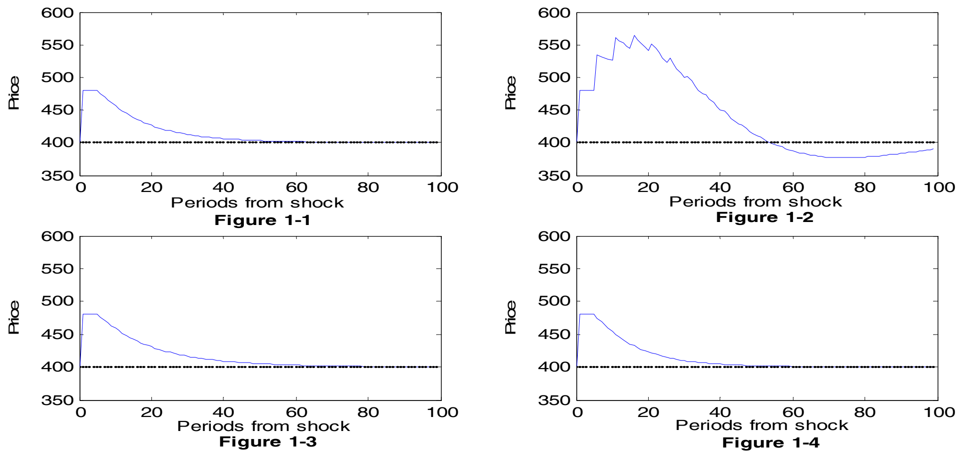

Figure 1 Market reaction to the interest rate shock in the heterogeneous expectation model (lag n=5; depreciation δ=0.05; demand elasticity=supply elasticity=1)

Figure 1-1 describes the dynamics of the naive expectation model using the above parameters. During an unanticipated shock of reduced interest rate, the demand increases accordingly. The price first rises quickly because of the delay of new supply. When the price reaches the peak, it will be kept at the maximum for several periods and then falling gradually when new space is supplied continuously. After some periods, the price approaches a steady state. Wheaton (1999) shows that with myopia, i.e. assuming the expected price at delivery time *t* is simply a constant capitalization of known rent at decision time *t-n*, the price trend is insensitive to the forthcoming supply. It will not lead to any over-built or under-built. Although he did not simulate the dynamics of other irrational expectation models, he pointed out that the backward-looking behaviour will have similar qualitative features to the one illustrated here. In this paper, the dynamics of some other irrational expectation models will be simulated. We shall give further analysis of a general difference equation to explain why the over-built or under-built is impossible for the given parameters.

Figures 1-3 and 1-4 describe the dynamics of the biased expectation and trend following expectation models respectively. They have similar qualitative features to those of the naive expectation model in consistence with the hypothesis of Wheaton. It is the parameter *h* in the biased expectation model that controls the decreasing rate of the price from the peak down to the steady state. With a higher *h*, the convergent rate is much slower. In contrast, with a smaller *h*, it converges more rapidly. It always approaches to the steady state gradually without oscillation. The dynamics of trend following expectation model is simulated in figure 1-4. It shows a sharp rise, reaching the peak and then falling to the steady state gradually. The change rate *g* controls the decreasing rate of the price from the peak to the steady state. Contrary to *h*, the higher *g* is, the higher the recovery rate will be. Similar to the naive expectation and biased expectation models, it approaches to the steady state gradually without passing the equilibrium point. Except when *h* and *g* are equal to 0, with both demand elasticity and supply elasticity equal to 1, equation (7) can be rewritten in a more general form,

$$x_{t+1} = \frac{ax_t}{1+bx_{t-k}} \qquad (15)$$

where, $a = \frac{1}{1-\delta}$, $b = \frac{\alpha_2}{1-\delta}$, $k = n-1$. Equation (15) is a discrete delay logistic equation of Pielou, where $a \in (1,\infty)$, $b \in (0,\infty)$ and $n \in Z^+$. Kuruklis and Landas (1992) proved that every positive solution of equation (15) oscillates about its positive equilibrium $(a-1)/b$ if and only if

$$\frac{a-1}{a} > \frac{(k)^k}{(k+1)^{k+1}} \qquad (16)$$

and when $k = 1$, every solution of the difference equation converges to the positive equilibrium. So the dynamics of the naive expectation model oscillates about the equilibrium point $p^* = \delta / \alpha_2$ if and only if

$$\frac{\frac{1}{1-\delta}-1}{\frac{1}{1-\delta}} > \frac{(n-1)^{n-1}}{(n)^n}, \quad \text{so} \quad \delta > \frac{(n-1)^{n-1}}{(n)^n} \qquad (17)$$

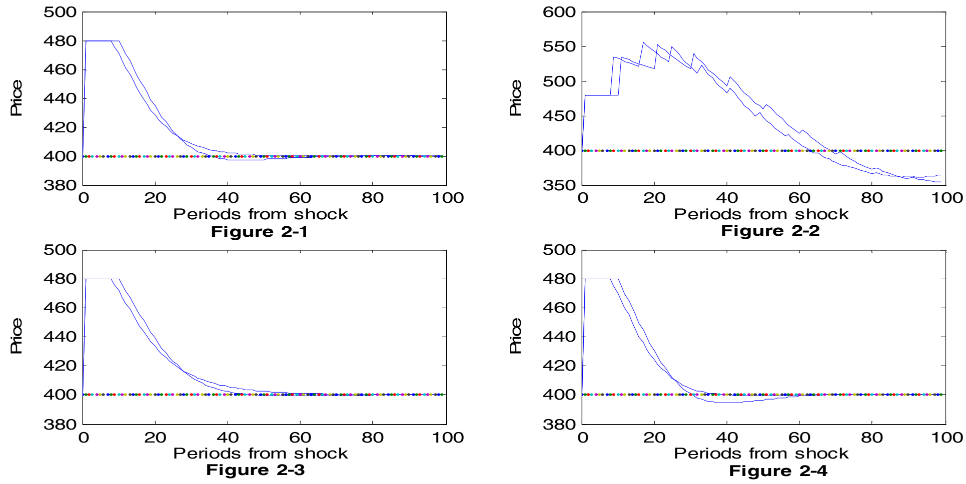

Figure 2 Market reaction to the interest rate shock in the heterogeneous expectation model as supply lag varied; depreciation δ=0.05; demand elasticity=supply elasticity=1

When δ=0.05 and *n*=5, the RHS of equation (17) is equal to 0.0819, which is larger than the LHS. So oscillation does not exist. But when the supply lag time is more than *n*=8, oscillation exists contradicting Wheaton's conclusion that the myopic model never displays oscillations if demand is at least as elastic as supply. Figure 2-1 shows the oscillating results of the model about its equilibrium point p^* with *n*=8 and *n*=10 respectively. The dynamics of biased expectation and trend following expectation models are qualitatively similar to the naive expectation model. Equations (12) and (14) can also be transformed into equation (15). They obey the rules of stability of Pielou's equation. Oscillations can occur in the biased and trend following expectation models with increasing *n*, as shown in figures 2-3 and 2-4.

Different properties of the adaptive expectation model are shown in figure 1-2 for *n*=5 and ω=0.3. Although new space is delivered after the supply lag time, price still goes up. It may be due to the fact that agents anchor their expectations on some initial values (Kahneman and Tversky 1979, 1982). For example, their previous forecast price falls into the initial neighbourhood called the basin of attraction. Although new space is delivered gradually, the expected price at delivery time will not fall due to the adaptive expectation arising from the persistent effects caused by the previous high price. A smaller ω implies that the forecast of price at delivery time depends more on the expected price than the actual price at decision time. After some alternating increase and decrease of price, the trend falls gradually despite some local increases during the falling process. Another difference is that the oscillation in figure 1-2 would not be possible in the other irrational expectation models when *n* is equal to or smaller than 5. The dynamics seems to be more active in the adaptive expectation model. Figure 2-2 shows that when *n* increases to 8 or even 10, such activity will be less energetic.

Figure 3 shows the dynamics of the heterogeneous expectation model when the demand elasticity is less than the supply elasticity, in particular for $\beta_1 = 0.4$ and $\beta_2 = 2$. Other parameters are fixed at $p^* = \$\,400$, *i*=5%, and *n*=5 years.

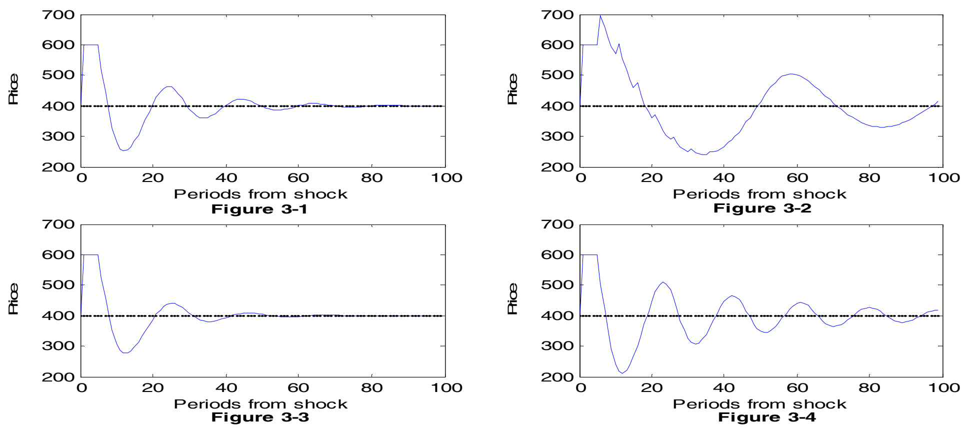

Figure 3 Market reaction to the interest rate shock in the heterogeneous expectation model (lag *n*=5; depreciation δ=0.05; demand elasticity β_1=0.4; supply elasticity β_2=2)

The dynamics of four irrational expectations models are shown. All are oscillating convergent for the given parameters. Figures 3-1, 3-3 and 3-4 describe the dynamics of the naive expectation, biased expectation and trend following expectation models respectively. They are qualitatively similar with different amplitudes. After the interest rate shock, the prices rise and keep high due to the lagged supply. Then prices go down and pass through the steady state when new space is delivered gradually.

Oscillation due to a bigger elastic supply than demand generates enough new construction momentum in consistence with Wheaton. The biased expectation model adopts a positive value of *h*. Estate agents are continuously optimistic to the expected price, oscillations are minimized. A negative *h* generates a reverse response and aggravates the oscillation amplitude. In the dynamics of the naive expectation and biased expectation models, oscillations decrease gradually and converge to the steady state after several periods. But the dynamics of the trend following expectation model generates more severe oscillation and converges to the steady state after a relatively longer period. Figure 3-2 describes the dynamics of the adaptive expectation model. With the given parameters, the price still goes up shortly after new space is delivered. Some local increase occurs during the first falling trend. The dynamics of the adaptive expectation model shows a lower frequency cycle than that of the other irrational expectation models.

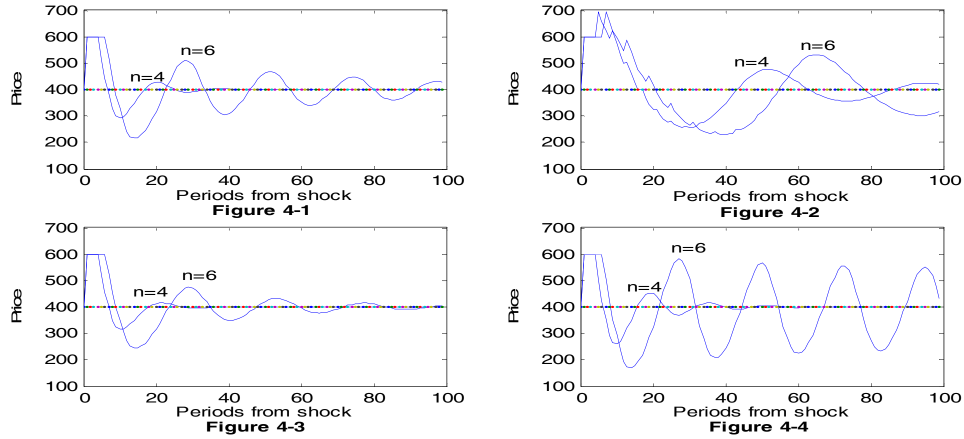

Figure 4 Market reactions to the interest rate shock with heterogeneous expectations as supply lag varied (depreciation δ =0.05; demand elasticity β_1 =0.4; supply elasticity β_2 =2)

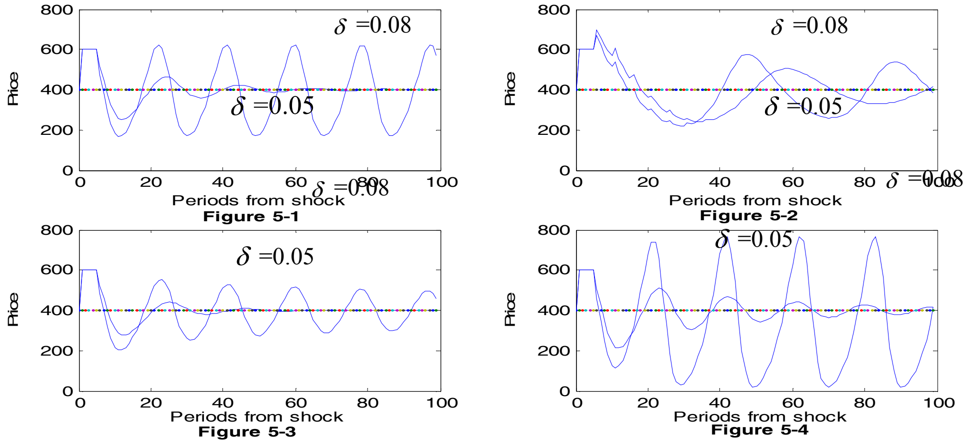

Figure 5 Market reactions to the interest rate shock with heterogeneous expectations as depreciation varied (lag *n*=5; demand elasticity β_1 =0.4; supply elasticity β_2 =2)

Figures 4 and 5 display the dynamics of the heterogeneous model as the parameter *n* and δ varied respectively. Although the increases of *n* and δ add instability to the model while keeping the main qualitative properties, there are some obvious differences. When supply lag time *n* increases from 4 to 6, the oscillation amplitude increases in all the four irrational expectation models. The convergence from the maximum or minimum to the steady state is still obvious, especially in the naive expectation and biased expectation models. When the depreciation rate δ increases from 0.05 to 0.08, the oscillation amplitude also increases. The convergence to the steady state is not very obvious. Another difference is the cycle frequency. When the parameter of supply lag *n* increases from 4 to 6, the frequency decreases in all four models with increased amplitude. When depreciation rate δ increases from 0.05 to 0.08, both the amplitude and the frequency increase.

CONCLUSIONS

Interest rate is one of the most popular financial instruments to adjust the real estate market. It seems very efficient in one place but really not in others. Therefore, each real estate market reacts uniquely during an unanticipated sudden change of interest rate.

This paper shows that housing price oscillation responds to some important factors, including demand elasticity and supply elasticity, supply lag and depreciation. That demand elasticity is less than supply elasticity is not the necessary condition for the occurrence of oscillation. Even when the demand elasticity is equal to supply elasticity, oscillation can be observed for long supply lag. When demand elasticity is less than supply elasticity, such oscillations turn to be more severe. The increase of either supply lag or depreciation rate can add instability to the oscillation. With the increase of supply lag, although the oscillation amplitude increases in all four irrational expectation models, the convergence from the maximum or minimum to the steady state is observed with decreasing cycle frequency. However, with the increase of depreciation rate, the oscillation amplitude shoots up and the departure from equilibrium attenuates very slowly with decreasing cycle frequency. Since the demand and supply elasticity, supply lag and depreciation are different in different real estate environments, it is natural for each estate market to have unique behaviour due to an interest rate shock.

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