ON THE MATHEMATICAL MODELLING OF WHOLE-LIFE COSTS

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Published mathematical whole-life costing (WLC) models are investigated to identify their strengths and weaknesses. Various models have been evaluated against five evaluation criteria including: scope of application, ease of implementation, computational effectiveness, transparency, and theoretical soundness. The first evaluation criterion look sat the applicability of WLC models in various whole life phases. The second criterion evaluates ease of implementation in terms of data requirements and ease of computer implementation. The third criterion assesses the computational efficiency of models to do basic WLC calculations. The fourth criterion assesses models in terms of clarity of definitions, the type of the cost breakdown structure and the ease of tracking WLC contributions of various cost elements. The fifth criterion looks at various assumptions and simplifications employed within various models. Results of the study have shown that most of published WLC models use the same basic equation. However, they differ in the breakdown of cost elements. Besides, each of these models seems to have some advantages and disadvantages regarding specific WLC applications. The paper concludes by discussion how existing models can be developed to offset their inherent weaknesses.

Keywords: financial management, whole-life costing, whole-life management, modelling.

INTRODUCTION

Whole-life costing (WLC) is a technique primarily used in the effective choice between a number of competing project alternatives. In a typical WLC exercise, the analyst employs an explicit mathematical model based on the discounted concept to calculate whole-life costs, normally as net present values (NPVs), of various alternatives. Although a WLC decision-making exercise can be done at any stage of the project, it is most beneficial during early design stages.

WLC can also be used as a management tool where the main objective is to assess and control costs throughout the whole life of the building to obtain the greatest value for the client (Flanagan et al., 1983; Seeley 1996). A related activity is whole-life costing analysis (WLCA) that aims to relate running costs and performance data and to provide feedback to the design team about the running costs of occupied buildings (Flanagan et al., 1989). According to Kishk et al. (2003a), effective WLC management requires the following capabilities (Kishk et al., 2003a):

- recording the actual performance and cost history of the building,
- analysis of the recorded history and feed-back of experience to the design stage,

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- assessment and control of costs throughout the whole life of the building, and
- planning the timing of work and expenditure.

In the last two decades, several mathematical models have been developed to support the above main WLC processes. These can be broadly classified into: decision-making models and management models. In this paper, these models are critically reviewed to reveal their advantages and disadvantages and to identify how they can be developed to offset their inherent weaknesses. First, decision-making models are examined. Then, management models including running cost models, significant cost models and refurbishment and replacement models are discussed. Finally, the work is summarised and direction for further research are introduced.

DECISION-MAKING MODELS

Most WLC models found in the literature employ the NPV approach. However, different nomenclature and/or cost breakdown structure are used to describe principal components of WLC. Generally, two broad categories can be identified. The first approach is based on discounted cash flow (DCF) modelling and the second approach is based on discounting costs based on their recurrence status.

DCF Models

Many researchers (e.g. Flanagan et al., 1989) have employed the following simple NPV formula based on the discounted cash flow (DCF) technique

$$NPV_i = \sum_{t=0}^{T} \frac{C_t^i}{(1+d)^t} \quad (1)$$

To use this formula, it is necessary first to express every cost, C, by a number of equivalent cash flows, C_t^i, over the analysis period, T. However, this may be computationally expensive. Besides, the contribution of each cost to whole life costs cannot be easily followed.

Recurrence models

Obviously, DCF-based models can handle single future costs and annual recurring costs directly. However, non-annual recurring costs, e.g. representing non-annual maintenance activities, cannot be handled directly. This is mainly because they are usually treated as a number of future one-off costs implying that their frequencies are certain. However, this is not always the case. In the other category, however, non-annual recurring costs can be dealt with directly without the need to express each cost to a number of equivalent cash flows. Besides, the uncertainties of the frequencies of these costs can be effectively handled. Therefore, these models are more appropriate when WLC is used as a decision-making tool.

Bromilow and Pawsey (1987) proposed a model as a generalisation of a previous model developed by Bromilow and Tucker (1983). This model is expressed as

$$NPV = C_{0i} + \sum_{i=1}^{n} \sum_{t=1}^{T} C_{it}(1+d_{it})^{-t} + \sum_{j=1}^{m} \sum_{t=1}^{T} C_{jt}(1+d_{jt})^{-t} - D(1+d_d)^{-T} \quad (2)$$

where

C_{0i} ≡ the procurement cost at time t=0, including development, design and construction costs, holding charges, and other initial associated with initial procurement;

C_{it} ≡ the annual cost at time t ($0 \le t \le T$), of function i ($0 \le i \le n$), which can be regarded continuous over time such as maintenance, cleaning, energy and security;

C_{jt} ≡ the cost at time t of discontinuous support function j ($0 \le j \le m$), such as repainting, or replacement of components at specific times.

d_{it} & d_{jt} ≡ discount rates applicable to support functions i and j respectively.

D ≡ the value of asset on disposal less costs of disposal; and

d_d ≡ the discount rate applicable to asset disposal value.

The main feature of this model is the classification of maintenance activities as non-annual recurring costs and those that remain continuous.

Sobanjo (1999) proposed a WLC model assuming that, all costs and values can be treated as either single future or annual costs. The model employs two discount factors, PW and PWA, to calculate the NPV, as follows

$$NPV = \sum C_0 + \sum F \cdot PWS + \sum A \cdot PWA \tag{3}$$

where

$$PWS = (1+d)^{-t} \tag{4a}$$

$$PWA = \frac{(1+d)^t - 1}{d(1+d)^t} \tag{4b}$$

Sobanjo’s model has the apparent advantage of being simple. Besides, it assumes that each cost type, e.g. initial, consists of the summation of a number of costs, which gives the analyst some flexibility. However, the model can handle only single future costs and annual costs. This means that non-annual recurring costs can only be treated as a number of single future costs which is a computationally expensive procedure. In addition, the frequencies of these costs must be assumed certain to determine the number of the recurrences of these costs.

The model developed by Kishk and Al-Hajj (2000) calculates the life cycle cost of an alternative i, as the net present value, of all costs and the salvage value of that alternative as

$$NPV_i = C_{0i} + \sum_{m=1}^{nno_i} PWO_{im} F_{im} + PWA \sum_{j=1}^{nar_i} A_{ij} + \sum_{k=1}^{nnr_i} PWN_{ik} C_{ik} - PWS \cdot SAV_i \tag{5}$$

where PWO_{im}, and PWN_{ik} are discount factors for, one-off non-recurring, and non-annual recurring costs, respectively, given by

$$PWO_{im} = (1+d)^{-t_{im}} \tag{6a}$$

$$PWN_{ik} = \frac{1-(1+d)^{-n_{ik} f_{ik}}}{(1+d)^{f_{ik}} - 1} \tag{6b}$$

$$n_{ik} = \begin{cases} \text{int}(\frac{T}{f_{ik}}), & \text{provided that rem}\left(\frac{T}{f_{ik}}\right) \neq 0 \\ \frac{T}{f_{ik}} - 1, & \text{elsewhere} \end{cases} \tag{6c}$$

This model has three unique features. First, a discount factor (equation 6b) was formulated to deal with non-annual recurring costs. Secondly an automatic expression for the number of occurrences of these costs has been derived (equation 6c). This expression accounts for the fact that non-annual costs recurring at the end of the last year of the analysis period are not taken into consideration. Thirdly, annual costs are assumed to be the summation of nar_i components, A_j, e.g. maintenance, operating and fuel costs. This was done to allow for more flexibility in the assignment of different uncertainty levels to various annual costs depending on the nature of every cost.

Based on this model, Kishk (2001) developed a model that calculates the life cycle cost of an alternative i, as an equivalent annual cost (EAC)

$$EAC_i = \sum_{j=1}^{nar_i} A_{ij} + AEI_i C_{0i} + \sum_{m=1}^{nno_i} F_{im} AEO_{im} + \sum_{k=1}^{nnr_i} C_{ik} AEN_{ik} - AES_i SV_i \tag{7}$$

where AES_i, AEI_i, AEO_i, and AEN_i are uniform annual equivalence factors for salvage value and initial, non-recurring, non-annual recurring costs, respectively. These factors are given by

$$AEI_i = \frac{d}{1-(1+d)^{-T_i}} \tag{8a}$$

$$AEN_{ik} = \frac{d\left(1-(1+d)^{-n_{ik} f_{ik}}\right)}{\left(1-(1+d)^{-T_i}\right)\left((1+d)^{f_{ik}} - 1\right)} \tag{8b}$$

$$AES_i = \frac{d}{(1+d)^{T_i} - 1} \tag{8c}$$

$$AEO_{im} = \frac{d\left(1+d\right)^{-t_{im}}}{(1+d)^{-T} - 1} \tag{8d}$$

This model has the same advantages of the previous model. Besides, the calculation of whole life costs as EACs is another merit when dealing with options with different lives as discussed earlier.

Kishk (2004) proposed the following normalised version of his earlier NPV model (Eq. 5).

$$N\hat{P}V_i = \hat{I}_i \left(1 + \sum_{m=1}^{nno_i} PWO_{im} \overline{F}_{im} + PWA \sum_{j=1}^{nar_i} \overline{A}_{ij} + \sum_{k=1}^{nnr_i} PWN_{ik} \overline{C}_{ik} - PWS \cdot S\overline{A}V_i \right) \tag{9}$$

where $\overline{F}_{im}$, $\overline{A}_{ij}$, $\overline{C}_{ik}$ and $S\overline{A}V_i$ are normalised variables given by

$$I_{0i} \cdot \overline{F}_{im} = F_{im} \tag{10a}$$

$$I_{0i} \cdot \overline{A}_{ij} = A_{ij} \tag{10b}$$

$$I_{0i} \cdot \overline{C}_{ik} = C_{ik} \tag{10c}$$

$$I_{0i} \cdot S\overline{A}V_i = S\overline{A}V_i \tag{10d}$$

$$I_{01} \cdot \hat{I}_i = I_{0i} \tag{10e}$$

$$N\hat{P}V_i = \hat{I}_i \cdot N\overline{P}V_i \tag{10f}$$

This model has been derived such that uncertainty of all input variables can be effectively modelled. Besides, the resulting whole-life costs are ratios of initial costs with a clear interpretation. Furthermore, it has two advantages over the standard NPV model (Eq. 5) when dealing with normalized data. First, it saves the time of preparing the data in the standard format. Secondly, and more importantly, the confidence measures in ranking can be better because of the elimination of the additional uncertainty in the predicted WLCs that may be caused by expanding normalised data to the standard format.

MANAGEMENT MODELS

Running cost models
Al-Hajj (1991) and Al-Hajj and Horner (1998) developed simple cost models to predict the running and maintenance costs in buildings. These models are based on the finding that for defined building categories identical cost-significant items can be derived using a statistical approach. Based on this analysis, a constant cost model factor is obtained. These models can be expressed in the form

$$R_c = \frac{1}{cmf} \sum_{i=1}^{n} \sum_{t=1}^{T} C_{(csi)it} (1+d)^{-t} \tag{11}$$

where

R_c	≡	the present discounted running costs over period T measured from time of procurement;
cmf	≡	cost model factor (constant for various building categories).
$C_{(csi)}$	≡	cost significant items: decoration, roof repair, cleaning, energy, management cost, rates, insurance and porterage.

Then, NPV can be calculated as (Al-Hajj, 1996):

$$NPV = C_O + \frac{1}{cmf} \sum_{i=1}^{n} \sum_{t=1}^{T} C_{(csi)it} (1+d)^{-t} - d(1+d_d)^{-T} \tag{12}$$

These models represent a significant simplification. Besides, they provide a rational framework for planning a minimum cost operations and maintenance strategy. However, their accuracy lie outside the expected range specified by Al-Hajj (1991) as revealed by the investigation carried out by Young (1992). She pointed out that these inaccuracies might be due to three reasons. First, the data recording system of one of the sources was different from the BMI-based coding system used in the development of the models. Secondly, these models do not take account of different materials or components being used in various buildings. Thirdly, occasional high cost items usually occur. The first two reasons were mentioned by Al-Hajj (1991) as limitations of his models. In addition, he employed the moving average technique to account for the third limitation.

However, there are three more shortcomings that seem to limit the generality of almost all existing running cost models. First, there is a seeming lack of reliable historic data sets to develop these models. Secondly, a simple data normalisation procedure ($£/m^2$) is adopted. This procedure does not yield accurate results (Kirkham et al., 1999) because it ignores other factors such as age, location, level of occupancy, and standards of operation and management. Thirdly, and more importantly, historic maintenance data, in terms of time and cost, represent only that which was affordable (Ashworth, 1999).

Cost significance models

Based on earlier work on cost significance (Poh and Horner, 1995; Horner and Zakieh, 1996), Al-Hajj and Horner (1998) defined a cost significant item as that item whose cost is greater than the mean item value. Kishk et al. (2003b) proposed to modify this relation such that a cost significant item is that whose cost is greater than or equal the mean item value. This modification is necessary because if all the items have the same cost, no item would be identified as significant using the original definition of Al-Hajj and Horner. For n cost items, I_i, this modified significance condition can be expressed as

$$S = \left\{ I_i \middle| C_i \geq \frac{\sum_{j=1}^{n} C_j}{n} \right\} \tag{13}$$

Dividing both sides in the above inequality by $\sum_{j=1}^{n} C_j$, relation 13 can be expressed simply as

$$S = I_i \middle| SR_i \geq ST \tag{14}$$

The modified relation, however, needs to be generalised such that significant elements and objects of a building can be identified as well. Kishk et al. (2003b) argued that the choice of a whole-life cost measure, e.g. the net present value, is logical to achieve best value. Besides, it should reflect any change of the significance of an item over the analysis period. Two measures that satisfy both requirements can be identified: the cumulative WLC contributions (CWLC) and the remaining WLC contributions (RWLC).

Kishk et al. (2003) argued that the remaining whole life costs measure should be used for the significance relation because it would reflect the future significance of the item. This is in line with the main objective of the planning process. More importantly, it allows managers the chance to influence and control future costs. The remaining WLC measure, $RWLC_i$, of an item is the summation of all discounted cash flows within the period from the present time, pt, to the end of the analysis period, T, i.e.

$$RWLC_i = \sum_{t=pt}^{T} PWS_t C_t^i \qquad (15)$$

$$SR_i = \frac{RWLC_i}{\sum_{j=1}^{n} RWLC_j} = \frac{\sum_{t=pt}^{T} PWS_t C_t^i}{\sum_{j=1}^{n}\sum_{t=pt}^{T} PWS_t C_t^j} \qquad (16)$$

Refurbishment/Replacement Models

The life expectancy of a building may be theoretically indefinite, if it is correctly designed and constructed and properly maintained throughout its life. However, in practice, this life is frequently shorter due to physical deterioration and various forms of obsolescence (Flanagan *et al.*, 1989). This view is supported by the opinions of Aikivuori (1996) and Ashworth (1996, 1999) who questioned the usefulness of scientific data because it is almost solely concerned with component longevity and not with obsolescence.

The main source of physical lifespan is normally manufacturer and suppliers data. However, their information may be described under ideal or perfect circumstances that rarely occur in practice or of a commercial nature (Ashworth, 1999). The factor method described in the ISO 15686 standard (BS ISO, 2000) allows an estimate of the service life to be made for a particular component or assembly in specific conditions. It is based on a reference service life (RSLC) and seven modification factors. The RSLC is defined as the expected service life in a well-defined set of in-use conditions that apply to that type of component or assembly) and a series of modifying factors that relate to the specific conditions of the case. The estimated service life (ESLC) is given by

$$ESLC = RSLC \times A \times B \times C \times D \times E \times F \times G \qquad (17)$$

Where

A	≡	quality of components factor.
B	≡	design level factor.
C	≡	work execution level factor
D	≡	indoor environment factor
E	≡	outdoor environment factor
F	≡	in-use conditions factor.
G	≡	maintenance level factor.

On the other hand, obsolescence may be functional, technological, or economic (BS ISO, 2000). Other obsolescence criteria may include social, environmental, legal and change of fashion or tastes (RICS, 1986; Ashworth, 1996). However, there is often an economic reason underlying such replacements, e.g. lettability of a building (BS ISO, 2000). Maintenance and operating costs of built assets increase with time because its elements, equipments and systems become older. Besides, both rental income and resale values of an asset decrease as its design and decorations become outdated.

Therefore, a choice of an ideal refurbishment/replacement cycle is necessary to maximize the asset value. This can be done using standard techniques of discounted cash flow. Obviously, this depends on the maintenance strategy, i.e. planned, failure-based or condition-based strategies.

Many researchers (e.g. Wong, 2000) have shown that the optimum replacement cycle, R_n, or the economic life, is given by

$$R_n = n \left| \frac{NPV_n}{(1-(1+d)^{-n})} \right. \text{is a minimum} \qquad (18)$$

This is equivalent to finding the net present value over one cycle, and then capitalising it as if it is a constant income to perpetuity. One of the drawbacks of this model is that it can only be solved numerically. Besides, all input data variables are assumed to be certain.

Ashworth (1999) pointed out that obsolescence relates to uncertain events. He analysed data about the estimated life expectancy of softwood windows from a RICS/BRE paper (RICS/BRE, 1992). The analysis shows a life expectancy of about 30 years, with a standard deviation of 22 years and a range of 1 to 150 years. Consequently, he concluded that it is not possible to select a precise life expectancy for a particular building component on the basis of this sort of information. This is mainly because important data characteristics, e.g. the reason for the variability of life expectancies, are not included.

Another category of replacement models lies within the fields of failure statistical analysis and stochastic dynamic programming where failure events are uncertain and are represented in terms of probabilities. These models are out of the scope of the current work and will be discussed in a future paper.

CONCLUSIONS AND THE WAY FORWARD

A critical review of published mathematical whole-life costing models has been carried out. These models can be classified into two main categories: decision-making models and management models.

Decision-making models can further be classified into DCF-based and recurrence models. Most of published WLC models use the same basic NPV equation. However, they differ in the breakdown of cost elements. Besides, each of these models seems to have some advantages and disadvantages regarding specific WLC applications. Recurrence models are more cost effective because most of these models employ automatic expressions for calculating the number of occurrences on non-annual recurring costs. Besides, compact expressions are formulated for various discount and annual equivalence factors. In addition, the contribution of each cost to whole-life costs can be easily followed. Furthermore, they are more suitable for decision-making situations including risk and uncertainty. However, these models may still entail some improvements. For example, including multiple discount and inflation rates and allowing for relative weights of importance regarding various initial and follow-on costs.

On the other hand, management models include running costs, cost-significance and replacement models. Almost all existing running cost models are too simple to be useful. This is mainly because they employ a simple data normalisation procedure, cost per unit area, which ignores other crucial factors such as age, size, height, location, level of occupancy, hours of use and standards of operation and management of buildings.

The concept of cost significance simplifies whole-life costing by reducing the number of cost items considered. However, these simple models have several shortcomings that seem to limit their generality. The two most significant limitations are the assumed linear cost-significance relationship and the lack of homogeneous and reliable whole-life costing data collection systems.

Almost all existing life expectancy and optimum replacement models are based on either physical deterioration or economic obsolescence of building elements. The successful implementation of these models has been hampered by the awkward obstacle of linking mathematical models with the context information of whole-life data and its natural uncertainty.

While there has been a substantial literature on theoretical WLC modelling and the treatment of data uncertainties, there has been rather little work on relating whole-life data and crucial characteristics of occupied buildings including age, size, location, occupancy profile, hours of use, and more importantly, maintenance strategies employed.

It is crucial to advance WLC modelling and the analysis of various facets of uncertainty in WLC data. It is believed that developing systems that would encourage systematic data collection from occupied buildings keeping much of the context of data is another fundamental requirement for breaking the vicious circle of practical whole-life costing implementation.

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