RISK ALLOCATION IN PFI PROJECTS

Hiroaki Yamaguchi[1], Thomas E. Uher[1] and Göran Runeson[2]

[1]*Faculty of the Built Environment, UNSW, Sydney, 2052, NSW, Australia,*
[2] *Dept. of Building, Newcastle University, Callaghan, 2308, NSW, Australia*

Private Finance Initiative (PFI) is a popular method of developing infrastructure projects but is characterized by a complex problem of risk allocation, which frequently causes conflicts among the stakeholders. This paper presents a conceptual model of risk allocation, developed for PFI projects, focusing on how costs and profit are allocated between the government client and the private PFI contractor. The model identifies feasible risk allocations, conditions of stakeholders' risk attitudes and assessments for financially freestanding PFI projects. It is concluded that the conceptual model developed in this paper provides a theoretical basis for risk allocation in PFI projects.

Keywords: insurance theory, private finance initiative, risk allocation.

INTRODUCTION

The aim of this paper is to develop a model for risk allocation, suitable for Private Finance Initiative (PFI) projects and to explore some theoretical implications of applying such a model.

PFI is defined as a mechanism through which the public sector can secure improved value for money (VFM) in partnership with the private sector (Treasury Task Force, 1997). From the viewpoint of risk allocation, PFI projects are categorized into three types (Private Finance Panel, 1995): (i) the financially free-standing type where risk is fully transferred to the contractor; (ii) the joint venture type where risk is fully transferred from the client with some client's contribution; and (iii) the type of services sold to the public sector where risk is shared rather than fully transferred.

Despite the knowledge of the roles that different project stakeholders perform under most commonly used forms of project delivery methods, risk allocation has been a persistent problem. The common criticism is that risks have been allocated inequitably and with little regard for the established principles of risk allocation as championed by Abrahamson (1974, 1982). Rather, there is a general tendency to shift risks downs the line to the general contractor irrespective of whether or not the general contractor is able to control them. The general contractor responds by pricing some of the risks into a contingency while transferring other risks further down the line to subcontractors, who are generally least equipped to control them. This practice is based on an erroneous assumption that by transferring it, clients may avoid paying the full price of the risk, when, in fact, they are likely to pay more than the full price. The general contractor and subcontractors will attempt to pass the impact of the risks back up the line to the client and may respond by lowering the quality standards if unable to obtain compensation in other ways. The outcome of this is cost and time overruns.

In the absence of a workable risk allocation model, risk allocation in PFI projects continues to be problematic although the stakeholders are both aware of and

experienced in the treatment of risk. Because PFI projects are so much more complex in their structure, they generally carry more risk than other commonly used delivery methods. They almost always require the client to provide some form of guaranteeing the anticipated level of demand for the services of the proposed facility, while the provider will normally guarantee a specified fee structure, all within a predetermined time frame.

The common approach adopted on PFI projects is to assume that there is a rational allocation of risk among the stakeholders that achieves a 'best value for money' (VFM) for the public client. The public client transfers risk to the private PFI contractor to motivate better performance while balancing the payment of the contingency or risk premium required by the contractor as shown in Figure 1. This approach attempts to achieve the optimal level of risk allocation, which leads to significant improvements in the VFM for PFI projects (Arthur Andersen and Enterprise LSE, 2000).
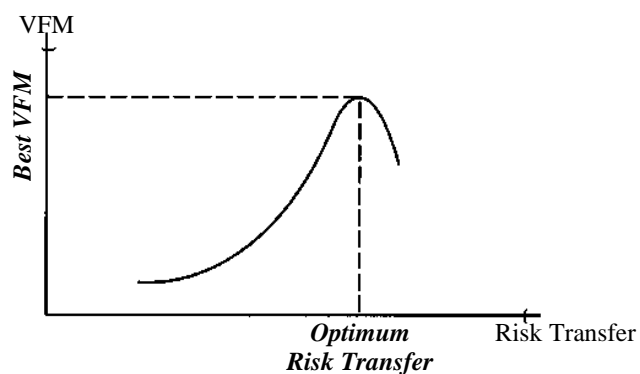

Figure 1: Rational allocation of risk (Treasury Task Force, 1997, p9)

A risk allocation problem concerns both qualitative issues (what type of risk is allocated and to whom) as well as quantitative issues (how much of the risk is allocated). In this paper we concentrate on developing a quantitative model for risk allocation in PFI projects. We firstly investigate existing approaches to risk allocation and adopt a quantitative approach to investigate the implications. Secondly, a conceptual model of risk allocation is developed with a linkage to the insurance theory. Finally the model is represented mathematically to extract theoretical implications.

RISK ALLOCATION APPROACHES

Abrahamson's principles of risk allocation (1974, 1982) prompted numerous studies of risk allocation between the client and the contractor in construction contracts (Mason, 1973; Ashley, 1977; Erikson, 1980; Porter, 1981). These studies followed two different paths. The first was qualitative and focused on empirical approach to risk allocation in contracts. The second approach was quantitative and focused on investigating theoretical implications in terms of the magnitude of the allocated risk.

Qualitative approaches to the risk allocation
Standardized forms of contract specify the obligations to be carried out by a party and some relief such as time extension for the party bearing the risk brought with the obligations. This approach provides a framework of risk allocation by a government

client based on the principle that each risk element should be distributed so that the effect on the total expected cost is minimized (Ashley, 1977, p19).

The qualitative approach leads to the development of the risk allocation matrix, which identifies what type of risk is allocated to whom. Numerous studies have investigated stakeholders' preferences about who bears what construction risks in most commonly used delivery methods (Erikson, 1980; Kangari, 1995; Snelgrove, 1994). Uher and Daveport (1998, p138) and NPWC/NBCC (1990) extend it further by developing a systematic method of risk allocation. The UK Treasury Task Force (1997) developed an example of the risk allocation matrix for PFI projects. In most cases PFI contractors must provide a risk allocation matrix when bidding (BDB, 1998; RTA, 1994). Arndt (1998), Arioka (1997) and PIARC (1999, p55) have investigated risk allocation in PFI projects with use of such a matrix.

These qualitative approaches are useful in practice as the risk matrix can directly help specify the relevant clauses in a contract. However, Levitt *et al.* (1979) pointed out that qualitative approaches are limited in addressing such issues as:

to what extent the parties share the various risk; and

how to rank possible strategies of risk allocation according to their impact on, for example cost, efficiency and satisfaction.

Quantitative approaches to the risk allocation problem

Quantitative approaches to risk allocation have been developed to extend the analysis to redress the limitations of the qualitative approach, especially the problem of how much of a risk is borne by each party. The quantitative approaches are based on the assumption that there is an optimal allocation of risk between the parties that can be achieved if the parties behave rationally (Levitt *et al.*, 1979, p3). Different aims and views of rationality lead to two different concepts of optimality: cooperative and competitive risk allocation.

Cooperative risk allocation assumes that the stakeholders jointly search for an agreement that is mutually acceptable. Most quantitative studies of risk allocation have defined the optimum solution as the allocation where the total contingency costs of the project are minimized, a target that in itself assumes cooperation between the stakeholders. Developments in decision sciences and computer programming have inspired research here. Using the axioms of cooperative game theory, Suijs (1999) and Suijs *et al.* (1999) have demonstrated that an allocation of the risk is Pareto optimal if- and only if-this allocation maximizes the sum of the certainty equivalents. For construction projects, Ashley (1977, 1980), Porter (1981) and McKim (1990) have developed models based on decision theory and identified the optimal level of risk allocation between the contractor and the client with computer simulation.

Competitive risk allocation, on the other hand, is the allocation where each of the stakeholders employs the strategy that best achieve their own goals without any concern for the other stakeholders. The model of competitive risk allocation developed by Levitt *et al.* (1979, 1980) considers the optimal risk allocation as the set of insurance policies, which minimizes the owner's insurance cost. Borch (1962, 1979, 1990) has, for example, proposed a model of risk allocation in marine insurance in a game theoretic framework.

Actual risk allocation mostly involves the mixture of cooperative and competitive allocations. This means that the solution gives room for negotiations. Potential

solutions together constitute the negotiation space. The importance of the negotiation space has been discussed in the emerging field of negotiation analysis (Sebenius, 1984; 1992), while Ashley (1977) has developed an approach to the negotiation for construction projects, which uses the Edgeworth box concept from economic theory.

The negotiation space has been formalized for premium negotiations by the insured and the insurer (insurance theory). By paying the premium (P), the person can change the risk or the form of the probability distribution $f(x)$, to fit to its risk preference. The feasible premiums satisfy the following relationships (Borch, 1962; 1990).

The insured: $\quad \int_0^\infty u(W - x)f(x)dx \le u(W - P)$ (1)

The insurer: $\quad \int_0^\infty u_1(W_1 + P - x)f(x)dx \ge u_1(W_1)$ (2)

where u is the utility function and W is the initial wealth. The equations specify the negotiation space on the utility space, given specific utility functions and probability distributions for the two parties.

A CONCEPTUAL MODEL OF RISK ALLOCATION

A combination of various risk causes, such as weather and inflation, result in risk events/outcomes, for example changes in cost, revenue, time and quality. A number of studies described the use of the risk matrix, which relates risk causes to risk events/outcomes (ICE, 1998). Monte-Carlo simulation is a popular tool for modeling various kinds of relationships in a quantitative manner (Vose, 2000).

Risk assessment is outside the scope of this paper. It is assumed that risk events/outcomes have been assessed in some manner. The paper focuses on the costs during construction and the profits during operation and maintenance (O&M). The problem of risk allocation is then the determination of the allocation ratios of the varied costs during construction and the varied profits during O&M, each of which is the deviation from the initially agreed estimates, and the risk premium associated with these allocation ratios. Each stakeholder decides the ratios and the premiums based on the assessment of the probability of the risk event/outcome.

Linkage to insurance theory
Each stakeholder has his/her own risk assessment based on the perception of the probability density function (PDF) with or without data and may assess the probability of a particular outcome x_1 as p_1. After the allocation with the ratio α, actual cost variation payable is αx_1 when the outcome x_1 occurs. So the stakeholder's assessment is tantamount to the assessment of the outcome αx_1 with probability p_1 (Figure 2). Assuming normal distribution with mean value 0 and standard deviation σ to the PDF before the allocation, standard deviation σ' for the post allocation becomes:

$$\sigma' = \sqrt{\frac{1}{n}\sum_i (\alpha x - 0)^2} = \alpha\sqrt{\frac{1}{n}\sum_i (x)^2} = \alpha\sigma.$$ (3)

Therefore, the stakeholder's assessment of risk after the allocation α becomes the PDF with standard deviation $\alpha\sigma$ (Figure 2).
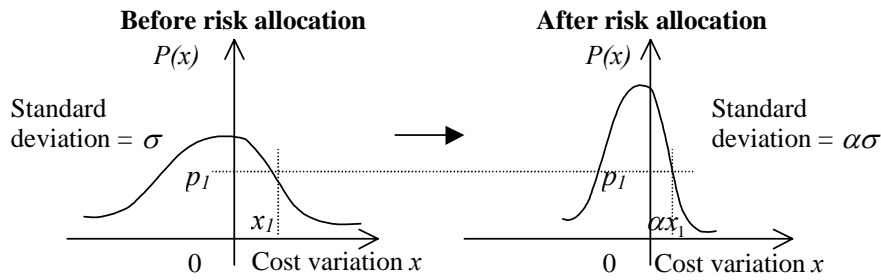
Before risk allocation

After risk allocation

Figure 2: PDF before and after allocation (allocation ratio: α)

This shows how stakeholders change their risk or the form of the PDF, which they face through the allocation of risks. This approach is similar to Ashley's risk allocation model (1977), which introduced the fraction of variance (σ^2) of the total cost as a proxy of the level of risk allocation. The model in this paper shows how the actual ratio in contracts can be converted to alter the form of the PDF. With this conversion, the actual allocation of the cost or profit variation in PFI contracts, which specify how much the risk is allocated, is linked to insurance theory, which regards risk allocation as change in the PDF.

RISK ALLOCATION MODEL OF PFI PROJECTS

This risk allocation model for PFI projects is an extension of Ashley's model (1977), which in turn is based on Borch's insurance theory (1962, 1990). Equation (1) and Equation (2) apply to the government client and the PFI contractor respectively. Both equations indicate that the situation after risk allocation is preferred to that before risk allocation according to the utility theory. It means that each stakeholder's expected utility after risk allocation is preferred to that before risk allocation. Table 1 summarizes the risk allocation between the two stakeholders during the construction and the operation and maintenance (O&M) phases.

Table 1: Costs and profits after risk allocation

Stakeholder	Wealth	Phase	Cost	Net profit
Government client	w_g	Constr.	$m + \tilde{x}\left[\sigma_g(1-\alpha)\right] + P$	-
		O&M	G	$(1-\beta)R + \tilde{y}\left[\delta_g(1-\beta)\right]$
PFI contractor	w_p	Constr.	$m + \tilde{x}\left[\sigma_p(\alpha)\right]$	$m + P$
		O&M	-	$\beta R + \tilde{y}\left[\delta_p(\beta)\right] + G$

where:

w_g	Initial wealth and the project value for the government client
w_p	Initial wealth and the fee without risk allocation for the PFI contractor
m	Cost estimate (assuming both parties have agreed the same estimate)
α	Allocation ratio of the risk of construction costs
β	Allocation ratio of the risk of O&M net profit
P	Risk premium or contingency cost for risk allocation of costs (α)
G	Government client's contribution to the contractor for the O&M service
R	O&M net revenue estimate (assuming both parties have agreed the same estimate)
$\tilde{x}[\sigma_i(\alpha)]$	Cost variation evaluated by stakeholder i with $N(0, \sigma_i(\alpha))$
$\sigma_i(\alpha)$	Standard deviation of the stakeholder i's risk evaluation of construction costs after the allocation α
$\tilde{y}[\delta_i(\beta)]$	O&M net profit variation evaluated by stakeholder i with $N(0, \delta_i(\beta))$

$\delta_i(\beta)$ Standard deviation of the stakeholder i's risk evaluation of O&M net revenue after the allocation β

Note that the method of risk allocation or profit sharing during the O&M phase assumes the simplest method where the risk allocation ratio (β) applies to the agreed estimate of profit (R) as well as the variation ($\tilde{y}$) from R.

The problem of risk allocation is the determination of risk allocation ratios (α and β) and risk premium including government client's contribution ($P + G$). The condition for the government client and the PFI contractor can be represented as:

$$\iint U_g\left(w_g - (m + \tilde{x}[\sigma_g]) + R + \tilde{y}[\delta_g]\right)f_g(x)g_g(y)dxdy$$
$$\leq \iint U_g\left(w_g - (m + \tilde{x}[(1-\alpha)\sigma_g] + P) + (1-\beta)R + \tilde{y}\left[\delta_g(1-\beta)\right] - G\right)f_{g'}(x)g_{g'}(x)dxdy \quad (4)$$

and

$$\iint U_p\left(w_p + m + P - (m + \tilde{x}[\alpha\sigma_p]) + \beta R + \tilde{y}\left[\delta_p(\beta)\right] + G\right)f_{p'}(x)f_{p'}(y)dxdy \geq U_p\left(w_p\right) \quad (5)$$

respectively, where U_i is the utility function of stakeholder i ($i = g, p$ where g is the government client and p is the PFI contractor), $f_i(x)$ and $g_i(y)$ is the PDF of stakeholder i before risk allocation of construction costs and O&M net revenue, respectively, and $f_{i'}(x)$ and $g_{i'}(y)$ is the PDF of stakeholder i after risk allocation of construction costs and O&M net revenue, respectively. Note that the risk during construction is independent to the risk during operation and maintenance (O&M).

RESULTS AND ANALYSES

For the purpose of simplification, let's assume that the stakeholder' utility function is expressed as:

$$U_i = -Exp(-a_i x) \quad (i = g, p),$$

and that the PDF for construction costs and net profits normally distributed as:

$$f_i(x) = \frac{1}{\sigma_i \sqrt{2\pi}} Exp[-\frac{1}{2}(\frac{x}{\sigma_i})^2] \text{ and } g_i(y) = \frac{1}{\delta_i \sqrt{2\pi}} Exp[-\frac{1}{2}(\frac{y}{\delta_i})^2] \quad (i = g, p), \text{ respectively.}$$

Feasible risk allocation
Equation (4) and (5) then result in:

$$P + G \leq \frac{a_g\left\{(2-\alpha)\alpha\sigma_g^2 + (2-\beta)\beta\delta_g^2\right\}}{2} - \beta R \quad (6)$$

and

$$P + G \geq \frac{a_p\left(\alpha^2\sigma_p^2 + \beta^2\delta_p^2\right)}{2} - \beta R \quad (7)$$

respectively.

The area bordered by these expressions constitutes the negotiation space for PFI projects. Figure 3 illustrates an example of the Edgeworth Box, given risk attitudes as $a_g = 0.1$ and $a_p = 0.3$, risk assessments as $\rho_g = 0.1$, $\rho_p = 0.3$, $\delta_g = 10$ and $\delta_p = 15$, and the net profit estimate $R = 60$, showing feasible risk allocation or combinations of α, β and $(P+G)$.

Figure 4 plots the negotiation space on expected utility space for various feasible combinations of (α, β) given certain feasible levels of the premium (P).
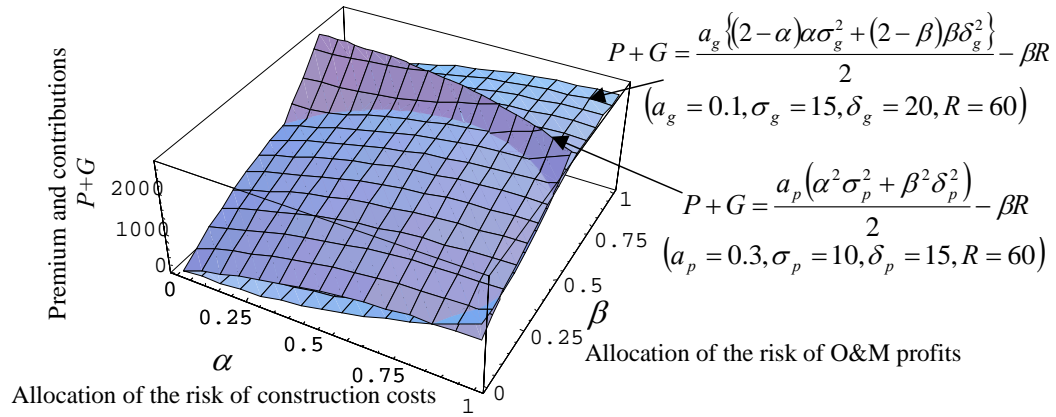
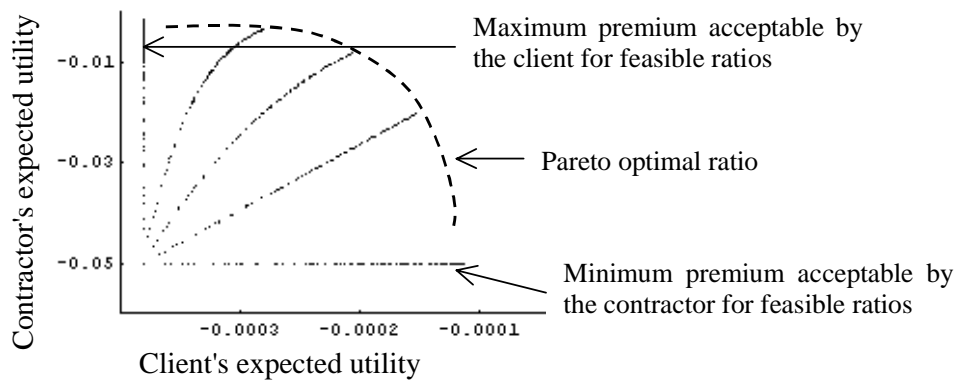**Figure 3:** Edgeworth Box for PFI projects

Figure 4: Negotiation space on the expected utility space

Conversely, given risk allocation, equations (6) and (7) can also give conditions of stakeholders' risk attitudes and assessments. For instance, the condition for a project to be feasible as a financially freestanding project, where all risks are transferred to the PFI contractor ($\alpha = 1$ and $\beta = 1$) with no government contributions ($G = 0$) is given in Appendix, Note 1 and yields:

$$a_g\left(\sigma_g^2 + \delta_g^2\right) \geq a_p\left(\sigma_p^2 + \delta_p^2\right).$$

Optimal risk allocation

Figure 4 shows the most efficient or Pareto-optimal ratios of risk allocation. Assuming α and β are independent, the ratios can be calculated from Jacobian condition as summarized in Appendix, Note 2. This yields:

$$\alpha = \frac{a_g\sigma_g^2}{a_p\sigma_p^2 + a_g\sigma_g^2} = \frac{1}{\left(\dfrac{a_p}{a_g}\right)\left(\dfrac{\sigma_p}{\sigma_g}\right)^2 + 1} \qquad \beta = \frac{a_g\delta_g^2}{a_p\delta_p^2 + a_g\delta_g^2} = \frac{1}{\left(\dfrac{a_p}{a_g}\right)\left(\dfrac{\delta_p}{\delta_g}\right)^2 + 1}$$

and

Note that this optimal allocation can only be achieved under the assumption that all stakeholders have complete information. In practice such ideal situations where all stakeholders reveal their risk attitudes and assessments are rare. However, this result can give such a 'benchmark' or 'best practice' to evaluate risk allocation in practice.

It may have been noted that the model does not determine the optimal premium and government contribution. Indeterminacy of the risk premium has been discussed in Raiffa (1968) and Borch (1979). Determination of the premium and the government

contribution requires additional axioms or the addition to the concept of 'fairness' into this model. Various types of cooperative game theory (Shubik, 1984; Thomson, 1994) and premium calculation principles (Goovaerts *et al.*, 1984) can give such solutions. Given risk allocations of actual projects in a country, the model can give such axioms or the country-specific convention of risk allocation deals.

CONCLUSIONS

The objective of this paper has been to develop a model for risk allocation, suitable for PFI projects and to outline some theoretical implications of applying such a model. The conceptual model linking risk allocation in contracts to insurance theory has been introduced and then the theoretical model has been developed based on Borch's insurance theory.

The model shows theoretical bases of risk allocation in PFI projects such as feasible risk allocation, conditions of stakeholders' risk attitudes and assessments of a certain type of PFI projects and optimal risk allocation under the complete information. The model may provide a benchmark of risk allocation and even a country-specific convention of risk allocation deals in PFI projects.

REFERENCES

Abrahamson, M.W. (1974) Contractual Risks in Tunnelling: how they should be shared. *Tunnels and Tunnelling*.

Abrahamson, M.W. (1982) Risk Management. *The International Construction Law Review*. 241-264.

Arioka, M. (1997) Risk Management in BOT Project for Public Infrastructures with an Example of the Sydney Harbour Tunnel Project. *The Journal of Risk Research*. **8**(1): 5-16.

Arndt, R.H. (1998) Risk Allocation in the Melbourne City Link Project. *The Journal of Project Finance*. 11-24.

Arthur Andersen and Enterprise LSE (2000) *Value for Money Drivers in the Private Finance Initiative*. London: HM Treasury.

Ashley, D.B. (1977) *Construction Risk Sharing*. Technical Report No.220, Dept of Civil Engineering, Stanford University.

Ashley, D.B. (1980) Construction Joint Ventures. *Journal of the Construction Division*. **106**(CO3): 267-280.

BDB (1998) *Call for Expressions of Interest*. Basslink Development Board, Hobart.

Borch, K.H. (1962) Equilibrium in a reinsurance market. *Econometrica*. **30**: 424-444.

Borch, K.H. (1979) Mathematical Models for Marine Insurance. *Scandinavian Actuarial Journal*. 25-36.

Borch, K.H. (1990) *Economics of Insurance*. Amsterdam: North Holland.

Erikson, C.A. (1980) *Risk Sharing in Construction Contracts*. PhD Dissertation, UMI.

Goovaerts, M., De Vylder, F. and Haezendonck, J. (1984) *Insurance Premiums*. Amsterdam: North Holland.

ICE (1998) *Risk Analysis and Management for Projects (RAMP)*. London: Thomas Telford.

Kangari, R. (1995) Risk Management Perceptions and Trends of US Construction. *Journal of Construction Engineering and Management*. (Dec.): 422-429.

Levitt, R.E., Ashley, D.B., Logcher, R.D. and Dxiekan, M.W., (1979) *A Quantitative Method for Analyzing the Allocation of Risks in Transportation Construction*. US Dept. of Transportation.

Levitt, R.E., Ashley, D.B. and Logcher, R.D. (1980) Allocating Risk and Incentive in Construction. *Journal of the Construction Division*. **106**(CO3): 297-305.

Mason, G.E. (1973) *A Quantitative Risk Management Approach to the Selection of Construction Contract Provisions*. Technical Report No.173, Dept of Civil Engineering, Stanford University.

McKim, R.A. (1990) *Risk Management for the Construction Owner*. PhD Dissertation, UMI.

NPWC/NBCC (1990) *No Dispute: Strategies for improvement in the Australian building and construction industry*. Report by NPWC/NBCC Joint Working Party.

PIARC (1999) *Financing and Economic Evaluation: Introductory Report*. XXIst World Road Congress, Kuala Lumpur.

Porter, C.E. (1981) *Risk Allocation in Construction Contracts*. Masters Thesis, UMI.

Private Finance Panel (1995) *Private Opportunity, Public Benefit*. London: HM Treasury.

Raiffa, H. (1968) *Decision Analysis: introductory lectures on choices under uncertainty*. Reading: Addison-Wesley.

RTA (1994) *Invitation for Preliminary Proposals to Finance, Design, Construct, Operate and Maintenance of Eastern Distributor Tollroad*. Sydney: Road and Traffic Authority.

Sebenius, J.K. (1984) *Negotiating the Law of the Sea.* Cambridge: Harvard University Press.

Sebenius, J.K. (1992) Negotiation Analysis: A Characterization and Review. *Management Science*. **38**(1): 18-38.

Shubik, M. (1984) *Game Theory in the Social Sciences: Concepts and Solutions*. Cambridge: The MIT Press.

Snelgrove, P.N. (1994) *Risk Allocation in Lump Sum Contracts*. Masters Thesis, UMI.

Suijs, J. (1999) *Insurance Game*. Center for Economic Research, Tilburg University.

Suijs, J., Borm, P. de Waegenaere, A. and Tijis, S. (1999) Cooperative games with stochastic payoffs. *European Journal of Operational Research*. **113:** 193-205.

Thomson, W. (1994) Cooperative Models of Bargaining, (eds. Aumann, R.J. and Hart, S.). *Handbook of Game Theory with Economic Applications. .***2.** Amsterdam: Elsevier.

Treasury Task Force (1997) *How to construct a Public Sector Comparator*. Technical Note No.5. London: HM Treasury.

Uher, T.E. and Daveport, P. (1998) *Fundamentals of Building Contract Management*. UNSW, 1998.

Vose, D. (2000) *Risk analysis: a quantitative guide*. New York: John Wiley.

APPENDIX

Note 1:
Given conditions for a project to be financially freestanding ($\alpha = 1$, $\beta = 1$, $G = 0$), equations (6) and (7) become:

$$P \le \frac{a_g\left(\sigma_g^2 + \delta_g^2\right)}{2} - R \quad (6') \text{ and } P \ge \frac{a_p\left(\sigma_p^2 + \delta_p^2\right)}{2} - R \quad (7'), \text{ respectively.}$$

For the project to be feasible, a positive premium (P) should exist. If the positive P exists, from (6') and (7') the following condition results:

$$\frac{a_g\left(\sigma_g^2+\delta_g^2\right)}{2}-R\geq P\geq\frac{a_p\left(\sigma_p^2+\delta_p^2\right)}{2}-R$$

$$\therefore a_g\left(\sigma_g^2+\delta_g^2\right)\geq a_p\left(\sigma_p^2+\delta_p^2\right)$$

Note 2:

Given β and G, Jacobian condition for α and P is written as:

$$\begin{vmatrix}\dfrac{\partial EU_g}{\partial\alpha} & \dfrac{\partial EU_g}{\partial P}\\[2mm]\dfrac{\partial EU_p}{\partial\alpha} & \dfrac{\partial EU_p}{\partial P}\end{vmatrix}=0$$

(a)

where EU_i is the expected utility of stakeholder i ($i=p,g$) after risk allocation.

For the government client, the expected utility after allocation is the right hand side of equation (4), and for the PFI contractor, that is the left hand side of equation (5), that is:

$$EU_g=-Exp\left[\frac{a_g}{2}\left[2(G+P+m-R+\beta R)-2W_g+a_g\left\{(\beta-1)^2\delta_g^2+(\alpha-1)^2\sigma_g^2\right\}\right]\right]$$

$$EU_p=-Exp\left[\frac{a_p}{2}\left\{2(G+P+\beta R)-2W_p+a_p\left(\beta^2\delta_p^2+\alpha^2\sigma_p^2\right)\right\}\right].$$

$$\therefore\quad\frac{\partial EU_g}{\partial\alpha}=-(\alpha-1)a_g^2\sigma_g^2 Exp\left[\frac{a_g}{2}\left[2(G+P+m-R+\beta R)-2W_g+a_g\left\{(\beta-1)^2\delta_g^2+(\alpha-1)^2\sigma_g^2\right\}\right]\right]$$

$$\frac{\partial EU_g}{\partial P}=-a_g Exp\left[\frac{a_g}{2}\left[2(G+P+m-R+\beta R)-2W_g+a_g\left\{(\beta-1)^2\delta_g^2+(\alpha-1)^2\sigma_g^2\right\}\right]\right]$$

$$\frac{\partial EU_p}{\partial\alpha}=-\alpha a_p^2\sigma_p^2 Exp\left[\frac{a_p}{2}\left\{2(G+P+\beta R)-2W_p+a_p\left(\beta^2\delta_p^2+\alpha^2\sigma_p^2\right)\right\}\right]$$

$$\frac{\partial EU_p}{\partial P}=-a_p Exp\left[\frac{a_p}{2}\left\{2(G+P+\beta R)-2W_p+a_p\left(\beta^2\delta_p^2+\alpha^2\sigma_p^2\right)\right\}\right]$$

Then, equation (a) is calculated as:

$$(\alpha-1)a_g^2\sigma_g^2\cdot a_p-a_g\cdot\alpha a_p^2\sigma_p^2=0$$

$$\therefore\alpha=\frac{a_g\sigma_g^2}{a_p\sigma_p^2+a_g\sigma_g^2}=\frac{1}{\left(\dfrac{a_p}{a_g}\right)\left(\dfrac{\sigma_p}{\sigma_g}\right)^2+1}.$$

The optimal β is obtained in the same manner.