PROJECT CASH FLOW FORECASTING: A MATHEMATICAL APPROACH

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A proactive approach to project cash flow management relies heavily on the use of a forecasting model that is, on the one hand, capable of generating reasonably accurate forecasts and, on the other hand, offers the flexibility which enables the financial manager to challenge the outcome of the forecast in the direction of corporate financial objectives of the organization. The latter was the subject of a paper in the previous ARCOM conference. This paper is concerned with the former: the structure of the project cash flow forecasting model.

The paper provides the details of the model, which consists of a mathematical expression, and a database of processed data relating to project expenditure flows of a variety of construction projects. The methodology used for the development of the model is based on the analysis of the shape of the project expenditure profile. The understanding of the shape-geometry of the profile and the identification of the variables that define the profile will in turn help to construct the profile for any given project definition. This undertaking also relies on the use of a mathematical model in order to convert the shape-variables into an expenditure pattern. Therefore, the paper discusses the components of the mathematical model, which consists of three modules each responsible for implementing a number of requirements relating to shape-variables.

Keywords: corporate financial management, financial management, mathematical models, project cash flow.

INTRODUCTION

The importance of cash flow forecasting and management has long been recognized by both practitioners and academics. The existence of many approaches to cash flow modelling and the variety of models within each category is on the one hand indicative of the importance of the subject area and on the other hand suggesting that a universal solution has not yet been offered. This is primarily due to the uncertainties relating to construction projects and project cash flow forecasting. The unpredictability associated with a project gives rise to the need for a proactive approach to risk management and risk appraisal.

In a presentation at a previous ARCOM conference the concept of risk was examined from a fresh perspective: instead of being considered as a “hazard”, to be avoided, transferred and contained, risk should be embraced as a potential source for gaining advantage over competitors. Moreover, the realization that risk could be a potential source of advantage led to the argument that further competitive edge could be gained by taking part in the creation of risk. (Khosrowshahi, 2000). With this perspective on risk, the paper examined construction project cash flow forecasting and management. It was argued that due to their varied priorities, the parties involved in the project may be able to engage in a negotiation in order to complement their priorities, thus the

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prospect of a win-win-win solution was considered as a realistic possibility. The paper examined the same from the contractor's perspective and outlined the stages whereby the contractor should undergo in order to prepare themselves for the negotiation. These stages are listed below:

1. Generate a forecast of project cash flow ("*what will happen"*)
2. Compare 1 with own corporate goals ("*what should happen"*)
3. Identify areas where compromise can be made to match 1 and 2.
4. For the client, identify areas for compromise and evaluate the extreme values.
5. Ditto for sub-contractors.
6. Commence with negotiations.

The practicality of this process was demonstrated by its application to a case study.

It is evident form the above stages that any attempt to match the project cash flow with the corporate objectives of the contracting organization, should initially commence with the generation of a realistic forecast of the cash flow. In other words, the actual cash flow should be known before it is altered in the direction of the corporate objectives. The current paper is concerned with this aspect of the work. Accordingly, the model that generates the forecast of project expenditure flow will be introduced in detail.

FORECASTING MODELS

There have been several models that can generate a forecast of project expenditure flow. Some of these models are based on elemental costing or activity-based costing. Accordingly, the elements of cost or the activities are identified and calculated along the scheduled timeline. These approaches are based on detailed examination of the sources of expenditure. Subsequently, in order to generate a forecast, these models require excessive resources which are not justified when contrasted against the predictive accuracy that they generate (Gunner and Betts, 1990). The alternative simplified approach consisted of the decomposition of areas of cost into labour, material, plant and overhead, and their breakdown into their respective time series (Harris and McCaffer, 1995). A more practical alternative has been based on the use mathematical models. Some of these have been based on the theoretical analysis of the behaviour of project expenditure flow (Hardy, 1970; Bromilow and Henderson, 1977 and Berny and Howes, 1982) and other on the analysis of data relating to past projects (Hudson 1978, Kaka and Price, 1993). Further work in this area has consisted of the use of artificial intelligence and expert systems (Brandon, 1988; Lowe *et. al.*, 1993; Boussabaine and Kaka, 1998 and Boussabaine and Elhag, 1999)

THE ALTERNATIVE APPROACH

The non-mathematical approaches to forecasting expenditure flow of construction projects offer a clear explanation as to the origin and nature of the resulting forecast, whereas, the mathematical approach to forecasting has the advantage of being simple, cheap and fast. These, mutually exclusive, features of different approaches to model development have lead this work to investigate the possibility of developing a mechanism for mathematical models to facilitate some degree of understanding about the behavior of project expenditure profile, hence, overcome the black-box nature of the mathematical solutions. This will allow taking advantage of the benefits of non-mathematical approaches without compromising the efficiency which is the characteristic of mathematical models. This undertaking has relied on the analysis of the shape of project expenditure patter.

The geometric approach to the analysis of project expenditure profile commenced by experienced individuals drawing a freehand sketch of 'S' curve of the likely shape of the profile. The curve normally consisted of three equal parts: front end, middle and back end. Cook and Jepson (1987) adopted the three-phased geometric approach to the curve generation and expressed it in a mathematical form. While the middle part is represented by a straight line, the front and back ends are generated by using a simple parabolic expression. The progress in this line of approach to analysis and simulation focused on improving the generation of the individual parts by exploiting elaborate mathematical expressions: Berny and Howes (1982), started with exploring the cubic polynomial: a curve is produced by one cubic curve enveloped within two identical curves produced by another cubic polynomial. Berny and Howes further expanded their approach to geometrical analysis by proposing a series of standard curves, offered visually, from which the most likely shape can be selected. At this point, the research work relating to the three-part approach to the generation of the shape geometry reached a terminal saturation. Further, these research works were ad-hoc attempts to explain the behaviour of the shape of expenditure profile and no effort has been made to establish links between shape-geometry and project variables.

The proposed work in this paper makes a conscious effort to analyse the profile from the shape-geometry perspective and refrains from limiting the possibilities to the three-part symmetrical structure. The promise of this work is that the variety of project definitions generate a variety of expenditure profiles and a rational explanation exists as to the relationship between the shape of the expenditure profile and the nature of projects. In this paper, the visualization of project expenditure pattern is not based on subjective assumptions about the project in its financial context, as the case is with Berny and Howes (1982). The latter requires the expert user to fully understand the project and select from a set of standard profiles. The proposed approach in this paper requires no subjective input from the expert and the visualization is based on objective evaluation of the relationship between shape variables and project definition. This is the product of a comprehensive set of statistical analysis (Khosrowshahi 1996).

THE PROPOSED EXPENDITURE FORECASTING MODEL

The research approach consisted of the following two stages:

1. Identification of the variables associated with the physical shape of the expenditure profile, namely, the shape-variables.
2. Development of a mathematical expression which, for a given set of shape variables could generate the pattern.

The identification of the shape-variables was based on the analysis of a large number of project data related to their expenditure profiles. However, prior to the analysis of the profiles, they had to be smoothed, in order to remove local fluctuations of successive payment figures. Several smoothing techniques were experimented but none provided a viable solution, thus an alternative smoothing method had to be devise. Subsequently, a new method was devised which proved to satisfy all the requirements for the smoothing process. The method consists of the following steps:

1) Running Median of 4th. Degree

2) Moving Average of 2nd. Degree

3) Running Median of 5th. degree

4) Running Median of 3rd. Degree

5) End Point Adjustment

6) Weighted Average; 0.25, 0.5 and 0.25.

7-14) Residual Values:

a. calculate the residual values,

b. smooth the residual values(stages a to f).

c. add smoothed values. to smoothed residual values.

The examination of the smooth profiles commenced with the observational analysis and resulted in the identification of a set of shape-variables. This result was then reconfirmed through the application of Principal Component Analysis (Khosrowshahi, 1996). These variables are discussed below.

1. As demonstrated in Figures 1 and 2, for each individual project, the periodic payment peaks to a particular position on both the time and value axis.

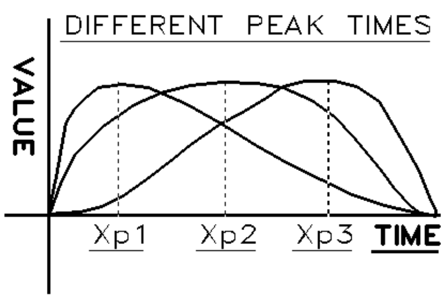

Figure 1: Varying peak value

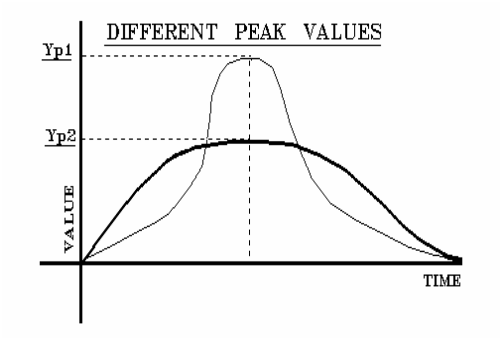

Figure 2: Varying peak time

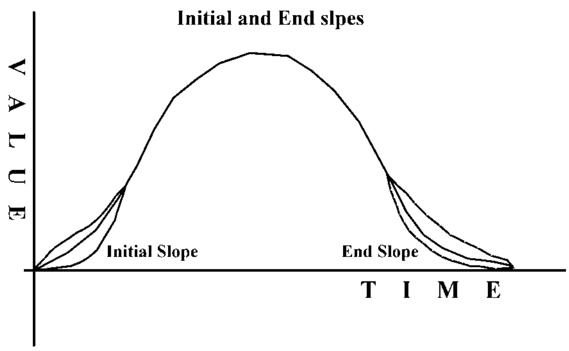

Figure 3: Initial & End Slope Examples

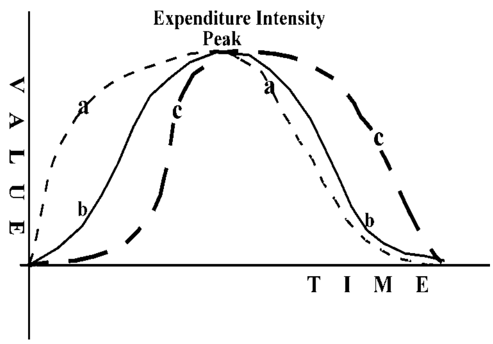

Figure 4: Examples of varying intensity

2. Different projects are also distinguished by the way their expenditure is developed at the beginning of the project and concluded towards the end of the project. These variations are measured and reflected in the values of the slope at the beginning and end of the profiles. An example is shown in Figure 3.

3. Another way by which the profiles of expenditure patterns are distinguished from one another is by the way the cumulative expenditures on both sides of the peak point are apportioned. This variable, referred to as the 'expenditure intensity', is closely related to the peak point period. Three possibilities are shown in Figure 4.
4. The examination of the smoothed profiles showed that the pattern often contains troughs as well as secondary peaks. These are referred to as 'distortion' the number of which tend to be less than 4 for each project. A distortion is measured in terms of its position (where the trough is), duration (the distance between the enveloped peaks), intensity (the total sum of values distorted) and type (accelerating or retarding). Figure five shows the variables of a distortion.

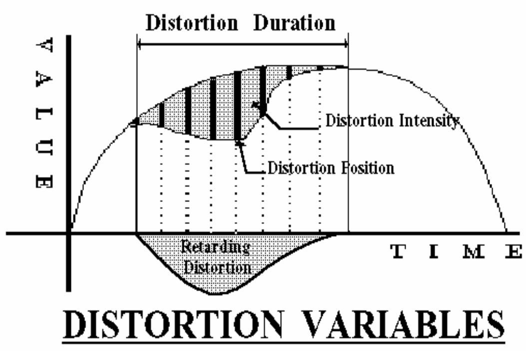

Figure 5: Variables of a Distortion

THE MATHEMATICAL MODEL

As part of the overall model, a mathematical model is required to convert the shape-variables into an expenditure profile. In the absence of an existing model, a new expression had to be developed which satisfied with the following requirements:

1. Comply with general characteristics of growth profile.
2. Comply with specific characteristics of the diversity of shape profiles.
3. Be independent of data [unlike methods like regression].
4. The parameters of the expression must be fully interpretable.

For an expression to satisfy all the requirements, it had to be divided into separate modules, each responsible for a particular set of requirements. Eventually, the following three modules were developed, 'control, Kurtosis and distortion'.

Control

While complying with the general properties of growth, this module should facilitate full control over the position of the peak on both the time and value axis. The curve produced by this module should have zero initial and end slopes so as not to interfere with other modules. Also, this module is responsible for controlling the overall area under the curve.

The requirement for the growth properties suggested that the module should be based on an exponential curve which satisfies the following requirements:

$$Y(0) = 0$$

Y(1) = 0
A parameter for controlling X_p
A parameter for controlling Y_p
A parameter for controlling the area under the curve

The mathematical expression for this module is:

$$Y_c = e^{bx^a(1-x)^d} - 1$$

$\boldsymbol{Y_c}$ represents the periodic value as a proportion of unity
x is the proportion of period no. over the no. of periods

Here

$$a = \frac{dR}{1-R} \qquad b = \frac{Log(1+Q)}{R^a(1-R)^d}$$

Where R and Q represent the peak time and peak value co-ordinates respectively.
The parameterization of *d* is carried out through numerical methods by using iteration.

Kurtosis

The kurtosis module is responsible for maintaining control over the value of the initial and end slopes as well as control over the value of curve-intensity.

The Polynomial expression was considered for this module. However, in order to maintain control over the behaviour of polynomial expression, particularly of a higher degree, it was decided to break the module into two parts: one starting from the origin to the peak point and other reversing from the end back to the peak point. Since there are five constraints, the degree of polynomial was selected to be 4, as follows:

$Y_k = k(X) = a_1X^4 + a_2X^3 + a_3X^2 + a_4X^1 + a_5$
Y_k : the periodic values for kurtosis
X: proportion of the period number over total kurtosis periods (origin to X_p or end to X_p)

The application of the constraints to the above equation provides the solution for the parameters as follows:

$a_5 = 0$
$a_4 = g$ [where *g* is the initial/end slope]
$a_3 = [h + g (3M - 2N - K)] / (N + L - 2M)$ [where h is the Exp. Intensity]
$a_2 = -(2a_3 + 3g)$
$a_1 = -(g + a_3 + a_2)$

Where

$N = X_1^4 + X_2^4 + X_3^4 + \ldots X_n^4$
$M = X_1^3 + X_2^3 + X_3^3 + \ldots X_n^3$
$L = X_1^2 + X_2^2 + X_3^2 + \ldots X_n^2$
$K = X_1^1 + X_2^1 + X_3^1 + \ldots X_n^1$

Therefore, the slope is identified by *g* and generated by the expression. As far as the curve intensity (*h)* is concerned, it is calculated as follows:

$$\sum_{i=0}^{i=Xp} Y_i = h = Na_1 + Ma_2 + La_3 + Ka_4 +$$

Distortion

The underlying pattern of growth is assumed to be smooth and continuous. However, often the pattern is distorted due to external abnormal events such as weather condition or sometimes due to the nature of the project. This phenomenon is referred to as *distortion.* As noted earlier, the parameters of a distortion are the position, duration, intensity and type. The expression used to simulate a distortion is basically similar to the Kurtosis module – a fourth degree polynomial. However, the constraints are somewhat different.

$Y_d(0) = 0$ zero initial state
$Y_d(1) = 0$ zero end state
$Y_d'(0) = 0$ zero initial slope
$Y_d'(1) = 0$ zero end slope

(slopes are maintained at zero, in order not to interfere with kurtosis module)

$$Y_d = d(x) = a_1X^1 + a_2 X^2 + a_3 X^3 + a_4 X^4 + k$$

Where Y_d is the value of distortion at proportional period X and k is a constant.
Assuming that the intensity of the distortion is $\boldsymbol{I}$, the following applies;

$$X_1^1 + X_2^1 + X_3^1 + \dots X_n^1 = K$$
$$X_1^2 + X_2^2 + X_3^2 + \dots X_n^2 = L$$
$$X_1^3 + X_2^3 + X_3^3 + \dots X_n^3 = M$$
$$X_1^4 + X_2^4 + X_3^4 + \dots X_n^4 = N$$

Then

$$a_1 K + a_2 L + a_3 M + a_4 N = I$$

The above are used to find the parameters *a1, a2, a3* and *a4* as follows;

$k = 0$
$a_1 = 0$
$a_2 = -(a_3 + a_4)$
$a_3 = -2a_4$
$a_4 = I / (N - 2M + 2L)$

Therefore, the intensity and duration of the intensity are generated through Y and X respectively. If the distortion is of retarding type then the values for Y are multiplied by (-1). The positioning of the distortion is where the second derivative of the equation is 0.

THE MODEL

The above mathematical modules can collectively generate the expenditure profile a given project where the values of shape-variables are known. Figures 6 and 7, shows the individual components of the periodic and cumulative patterns of expenditure for a project with the following shape parameters.

The identification of the shape variables and the development of the mathematical model, as described in this paper, constitute the major part of the overall research. However, what remains to be considered is the estimation of each shape-variable for a given project definition. This undertaking has been accomplished through the development of shape-variable estimation models (Khosrowshahi, 1996). In other words, for a given project definition, these models estimate the values of shape-variables before they are passed to the mathematical expression for processing. Currently there are over 78 such models for several types of project. These are the product of over 20,000 multiple regression analysis. Upon the availability of related data, additional models can be easily developed and incorporated into the overall model.

X_p	Y_p	Initial Slope	End Slope	Expend. Intensity	Distortions(1)

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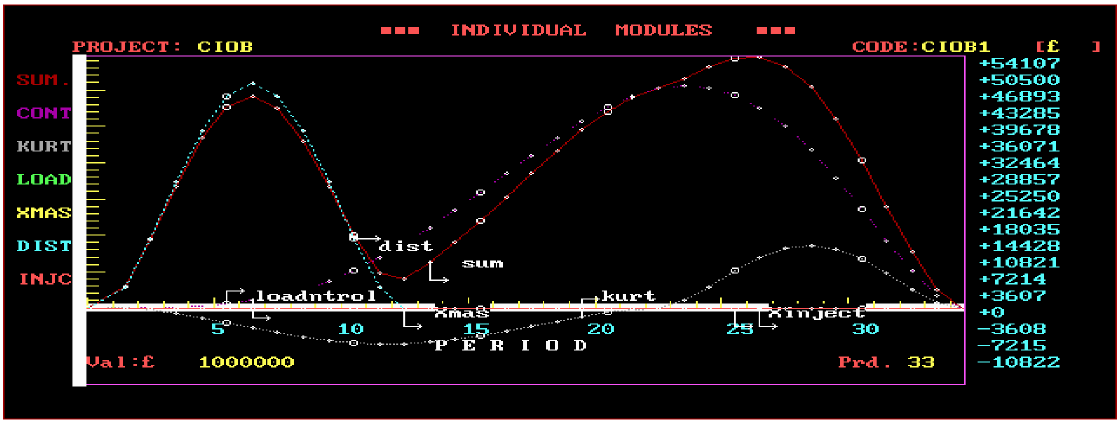

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Figure 6: Periodic Pattern of Expenditure: Individual Modules

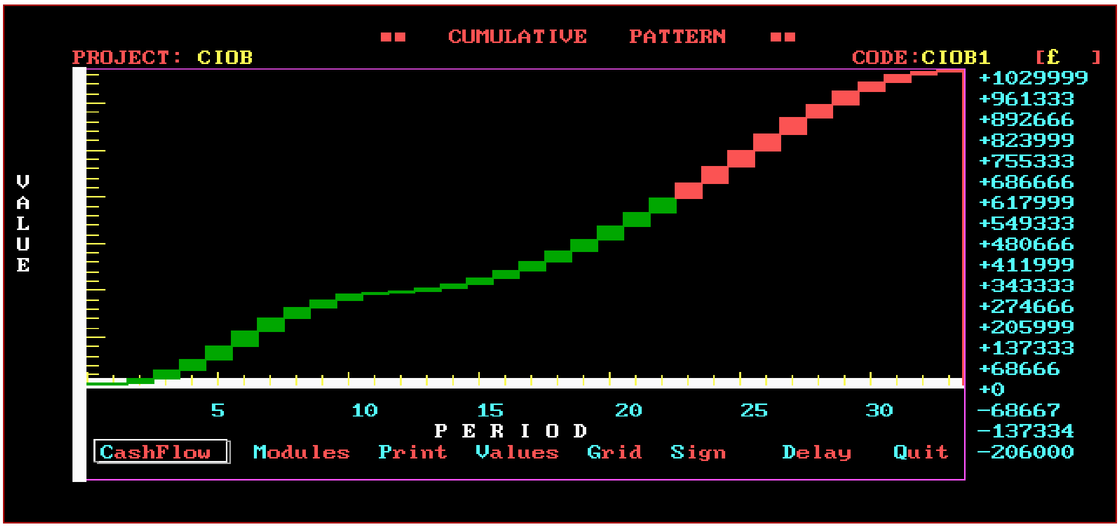

Figure 7: Cumulative Pattern of Expenditure

CONCLUSION

Previously, it had been argued that, due to the importance of project finance, all parties stand to benefit from negotiating the issues pertaining project finance. However, this process relies, initially, on the use a project expenditure forecasting model. The forecast of *what is likely to happen*, which is generated by the model, can form the basis for the negotiation. This paper has presented the details of a mathematical model for project financial forecasting and management.

It has been established that curve-geometry can provide a useful tool for analysing the behavior of project expenditure pattern. Subsequently, it was shown that the expenditure profile of construction projects could be defined in terms of a number of variables, namely shape-variables. These variables have found to be associated with the inherent features of the project. Also, a mathematical expression has been introduced which, for a given value of shape-variables, generates the expenditure profile. This mathematical expression consists of three independent modules each responsible for satisfying a number of requirements. While the ‘control module’ is an exponential function, representing the growth nature of the expenditure profile the ‘kurtosis’ and ‘distortion’ modules use two separate 4^{th} degree polynomials to control the peakedness of the profile and simulate the effects of distortions to the underlying pattern.

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