FUZZY MODELLING OF LIFE CYCLE COSTS OF ALTERNATIVES WITH DIFFERENT LIVES

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The necessity of applying fuzzy set theory to life cycle costing (LCC) is highlighted. A computer algorithm for LCC-based decision-making is outlined. This algorithm is based on the fuzzy set theory (FST) and interval analysis. The algorithm is designed around an explicit analytical LCC model. The model was introduced in a form that allows the handling of uncertainties in all input variables for alternatives with different lives.

The computer implementation of the model was carefully investigated. The solution of a selected example problem is also included. Analysis results confirm that the proposed model is powerful and transparent. In addition, it is shown that handling of uncertainties within the algorithm is superior to conventional risk assessment techniques.

The proposed model is the second in series being developed by the authors to overcome some of the difficulties in the implementation of LCC in the industry. These models will be integrated in a user-friendly LCC-based decision support system.

Keywords: decision-making, fuzzy set theory, interval analysis, life cycle costing, risk assessment.

INTRODUCTION

Kishk and Al-Hajj (2000a) carried out a critical review of risk assessment techniques that usually used in LCC-based decision-making. The sensitivity analysis does not provide a definitive method for decision-making. Besides, it is only useful when the uncertainty in one variable is predominant. On the other hand, simulation methods have been criticised for their complexity and their expense. Other simplified probabilistic methods were found to lack the generality of application. Besides, a major flaw in almost all of these methods is that they follow the characteristics of random uncertainty. This implies that significant historic data should be available to produce a statistically meaningful analysis. However, historic data for construction are too small (Edwards and Bowen, 1998) and more importantly are unreliable (Bull, 1993).

In view of the limited availability of 'hard data', subjective assessments for the likely values of uncertain variables have to be elicited from appropriate experts (Byrne, 1996, Clemen and Winkler, 1999). Even if historic data are available, it is common to adjust historic-based assessments with subjective opinions (Sobanjo, 1999). This seems to be inevitable in LCC analyses because historic data will never provide a precise solution and high quality judgment will always be required (Ashworth, 1996).

Kishk, M and Al-Hajj, A (2000) Fuzzy modelling of life cycle costs of alternatives with different lives. *In:* Akintoye, A (Ed.), *16th Annual ARCOM Conference*, 6-8 September 2000, Glasgow Caledonian University. Association of Researchers in Construction Management, Vol. 2, 577-86.

Kaufmann and Gupta (1988) criticised the use of deterministic and probabilistic mathematical tools in modelling socio-economic processes. The main criticism that socio-economic systems are subjected to human subjectivity both in measurement and decision-making. They described how to manipulate fuzzy numbers in the discounting problem using an approximate approach. Sobanjo (1999) employed this simplified approach to introduce a methodology for handling the subjective uncertainty in LCC analyses. Although this model is simple, it has two limitations. First, the interest rate and all time horizons were assumed to be certain. Moreover, only triangular fuzzy numbers (TFNs) were considered. However, an expert should give own estimates using the most appropriate membership function (MF) for every state variable.

In Kishk and Al-Hajj (2000a, 2000b), a powerful algorithm based on the fuzzy set theory (FST) and interval analysis was designed. This algorithm overcomes various limitations of Sobanjo methodology. In this paper, an LCC model that can deal effectively with subjective assessments of input variables of alternatives with different lives is developed. The model presented in this paper is in part an extension of the previous model. First, non-recurring (one-off) future costs are included in the model. Then, the model is formulated such that it can handle alternatives with different lives. Symbols used in the paper are listed in an appendix.

FORMULATION OF THE MODEL

The model proposed by Kishk and Al-Hajj (2000b) calculates the net present value of an alternative *i*, with fuzzy input data as

$$N\tilde{P}V_i = \tilde{C}_{0i} + \frac{1}{\tilde{r}}\left(1-(1+\tilde{r})^{-\tilde{T}}\right)\sum_{j=1}^{nar_i}\tilde{A}_{ij} + \sum_{k=1}^{nnr_i}\tilde{C}_{ik}\,\frac{1-(1+\tilde{r})^{-\tilde{n}_{ik}\tilde{f}_{ik}}}{(1+\tilde{r})^{\tilde{f}_{ik}}-1} - \left(1+\tilde{r}\right)^{-\tilde{T}}\tilde{S}_i \tag{1}$$

where

$$\tilde{n}_{ik} = \begin{cases} \operatorname{int}\left(\dfrac{\tilde{T}}{\tilde{f}_{ik}}\right), & \text{provided that} \quad \operatorname{rem}\left(\dfrac{\tilde{T}}{\tilde{f}_{ik}}\right) \neq 0 \\ \dfrac{\tilde{T}}{\tilde{f}_{ik}} - 1, & \text{elsewhere} \end{cases} \tag{2}$$

Assuming there are nno_i, non-recurring one-off future costs, $\tilde{F}_{im}$, occurring at times $\tilde{t}_{im}$. These costs can be discounted using the present worth factor, $P\tilde{W}S_{im}$ (Kirk and Dell' Isola, 1995), given by

$$P\tilde{W}S_{im} = (1+\tilde{r})^{-\tilde{t}_{im}} \tag{3}$$

Thus, the net present value of an alternative *i*, whose life cycle is T_i, may be expressed as

$$N\tilde{P}V_i = \tilde{C}_{0i} + \sum_{m=1}^{nno_i}\left(1+\tilde{r}\right)^{-\tilde{t}_{im}}\tilde{F}_{im} + \frac{1}{\tilde{r}}\left(1-(1+\tilde{r})^{-\tilde{T}_i}\right)\sum_{j=1}^{nar_i}\tilde{A}_{ij} + \sum_{k=1}^{nnr_i}\tilde{C}_{ik}\,\frac{1-(1+\tilde{r})^{-\tilde{n}_{ik}\tilde{f}_{ik}}}{(1+\tilde{r})^{\tilde{f}_{ik}}-1} - \left(1+\tilde{r}\right)^{-\tilde{T}_i}\tilde{S}_i \tag{4}$$

When comparing alternatives with different lives, a residual value is usually attributed to cover the remaining years. However, this can be arbitrary as pointed out by Flanagan *et al.* (1989). They recommended the use of the equivalent annual cost method. This method converts all costs to an equivalent uniform annual cost, $E\tilde{A}C_i$, using the present worth of annuity factor, $P\tilde{W}A_i$, as:

$$E\tilde{A}C_i = \frac{N\tilde{P}V_i}{P\tilde{W}A_i} \tag{5}$$

where

$$P\tilde{W}A_i = \frac{1}{\tilde{r}}\left(1-(1+\tilde{r})^{-\tilde{T}_i}\right) \tag{6}$$

Substituting Eqs. (4 and 6) into Eq. (5) and making some simplifications, the $E\tilde{A}C_i$ can be expressed as

$$E\tilde{A}C_i = \sum_{j=1}^{nar_i}\tilde{A}_{ij} + A\tilde{E}I_i\tilde{C}_{0i} + \sum_{m=1}^{nno_i}\tilde{F}_{im}A\tilde{E}O_{im} + \sum_{k=1}^{nnr_i}\tilde{C}_{ik}A\tilde{E}N_{ik} - A\tilde{E}S_i\tilde{S}_i \tag{7}$$

where $A\tilde{E}S_i$, $A\tilde{E}I_i$, $A\tilde{E}O_i$, and $A\tilde{E}N_i$ are uniform annual equivalence factors for salvage value and initial, non-recurring, non-annual recurring costs, respectively. These factors are given by

$$A\tilde{E}I_i = \frac{\tilde{r}}{1-(1+\tilde{r})^{-\tilde{T}_i}} \tag{8}$$

$$A\tilde{E}N_{ik} = \frac{\tilde{r}\left(1-(1+\tilde{r})^{-\tilde{n}_{ik}\tilde{f}_{ik}}\right)}{\left(1-(1+\tilde{r})^{-\tilde{T}_i}\right)\left((1+\tilde{r})^{\tilde{f}_{ik}}-1\right)} \tag{9}$$

$$A\tilde{E}S_i = \frac{\tilde{r}}{(1+\tilde{r})^{\tilde{T}_i}-1} \tag{10}$$

$$A\tilde{E}O_{im} = \frac{\tilde{r}\left(1+\tilde{r}\right)^{-t_{im}}}{(1+\tilde{r})^{-\tilde{T}}-1} \tag{11}$$

COMPUTER IMPLEMENTATION

The proposed model was implemented in the form of a computer algorithm. Crucial issues regarding the computational efficiency and robustness of the algorithm were carefully investigated. These issues are discussed in the following subsections.

Computer representation of fuzzy sets

Three methods of fuzzy set representation can be identified (Kaufmann and Gupta, 1988; Turksen, 1991; Ross, 1995): functional; paired; and level-set methods. The first method is limited to simple cases as many problems can arise when fuzzy sets are being combined. In a paired representation, a fuzzy set is defined by pairs (μ_i, x_i) in its discretized domain. The use of this method may lead to erroneous results (Dong *et al.*, 1985). On the other hand, the level-set method describes a fuzzy set by its α-level sets or simply α-cuts. This representation is robust and computationally effective. In addition, it eliminates problems associated with other methods (Ross, 1995). Moreover, it leads to a more understanding of the decision making process as will be shown later.

Method of Implementation

Zadeh (1975) developed a general method for extending mathematical concepts to deal with fuzzy quantities. This method is known in the literature as the extension principle. The implementation of the extension problem is difficult and consequently, the most appropriate method for implementing it should be identified (Ross, 1995). The restricted DSW algorithm (Givens and Tahani, 1987) and the vertex method Dong and Shah (1987) are two powerful methods that are based on the α-cut concept. The first method is valid only for non-zero positive intervals; while the latter is very effective for computations including multiple occurrences of identical interval numbers. A thorough investigation of Eqs. (7-11) reveals that equivalence factors (Eqs. 8-11) are best computed by the vertex method, while the restricted DSW method is more appropriate for all other computations.

Ordering of alternatives

Because the output of Eq. (7) is a fuzzy set, a fuzzy ranking method has to be included in the algorithm. A simple and effective method known as the area compensation (Fortemps and Roubens, 1996) was chosen. The method is based on introducing a function *R*, that maps the set of fuzzy quantities to the real line and to use natural ordering. Kaufmann and Gupta (1988) called this function ‘the removal’. If two fuzzy quantities have the same removal, they are ordered according to their mode. If the two quantities still have the same mode, they are ordered according to their divergence.

The algorithm

Based on the issues discussed above, the following computational algorithm may be proposed (Fig. 1).

Experts express their assessments of uncertain state variables as fuzzy quantities. These assessments are drawn with solid lines in Fig. (1).

Select an α value such that $0 \leq \alpha \leq 1$.

Find the interval in the discount rate, $\tilde{r}$, that correspond to this α.

Find the corresponding α-cut in the alternative life, $\tilde{T}_i$.

Use the vertex method to find the corresponding intervals in the factors $A\tilde{E}I_i$ and $A\tilde{E}S_i$ (Eqs. 8 and 9).

Find the intervals in the input membership functions for each non-annual recurring cost, $\tilde{C}_{ik}$, and its frequency, $\tilde{f}_{ik}$, corresponding to the chosen α.

Use the restricted DSW algorithm to find the interval in the membership function n_{ik}, for the selected α-cut level from Eq. (2). Then, use the vertex method to find the interval in the $A\tilde{E}N_{ik}$ factor for the selected α-cut level using Eq. (9).

Repeat steps 6 and 7 for all non-recurring costs for alternative *i* (nnr_i times).

Find the intervals in the input membership functions for each non-recurring cost, $\tilde{F}_{im}$, and its occurrence time, $\tilde{t}_{ik}$, corresponding to the chosen α.

Use the vertex method to find the interval in the $A\tilde{E}O_i$ factor for the selected α-cut using Eq. (11).

Find the intervals in the input membership functions for annual-recurring costs, $\tilde{A}_{ij}$, and using the restricted DSW algorithm to find their summation.

Find the interval in the input membership function for the salvage value, $\tilde{S}_i$.

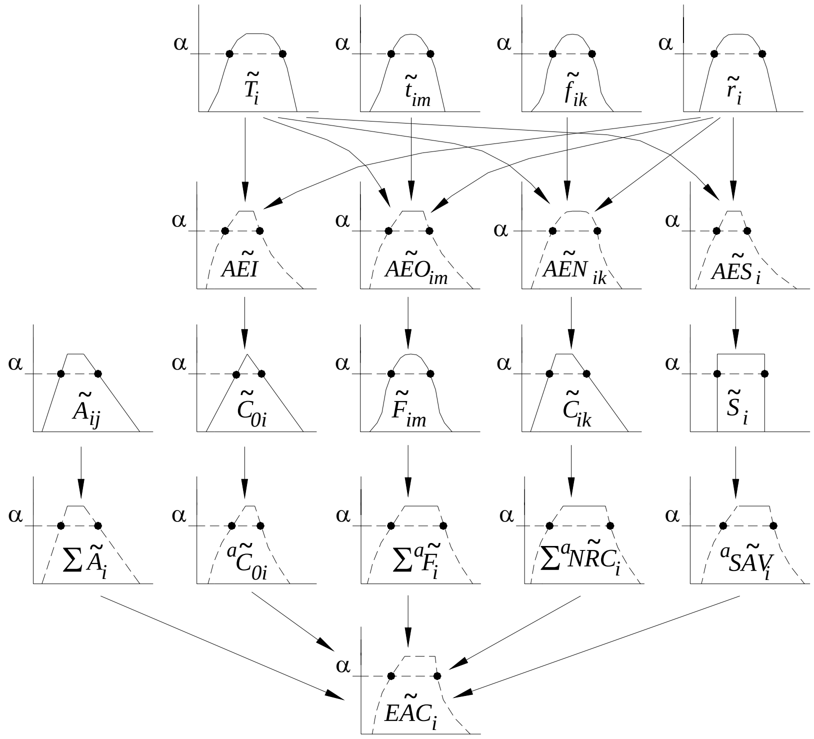


Figure 1: Schematic representation of the algorithm**.**

Using the restricted DSW algorithm, compute the interval for the output membership function for the selected α-cut level using Eq. (7).

Repeat steps 2 to 13 for different values of α to complete an α-cut representation of the equivalent annual cost for alternative *i*, $\tilde{EAC}_i$.

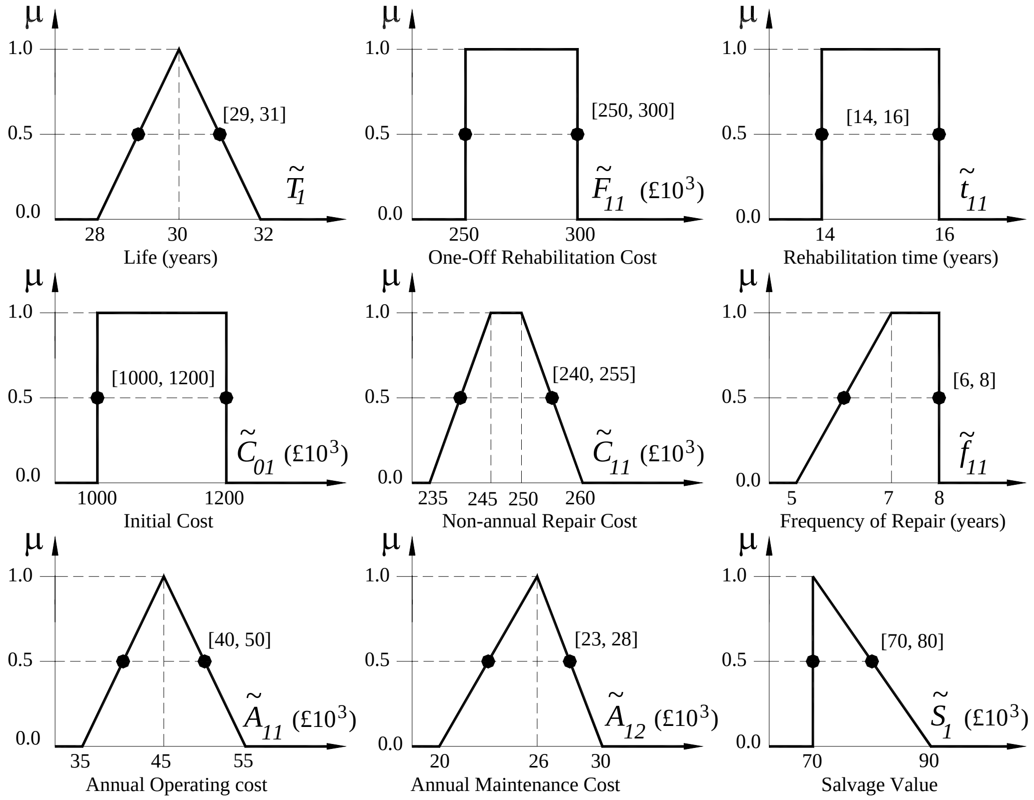
Repeat steps 2 to 14 for all alternatives.

Alternatives are ranked according to their net present values using the ranking procedure outlined in the previous subsection.

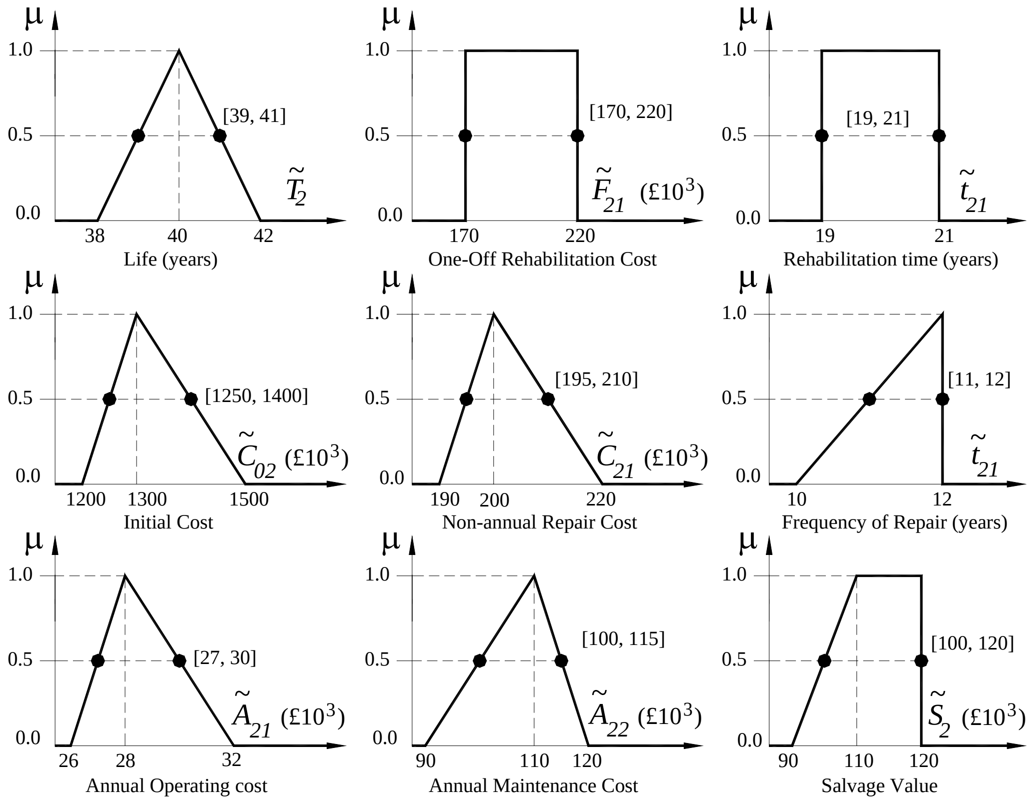
AN PROBLEM

In this section, the efficiency of the algorithm is explained in the context of an example problem. Figures (2 and 3) show the membership functions of the input variables for two competing design alternatives and the discount rate, respectively. The two alternatives are to be ordered according to their life cycle costs.

As can be seen in figures (2 and 3), input variables were assigned different MFs to reflect their uncertainty. By using an α-cut on any MF, an interval is derived. The value of α represents the expert's degree of confidence in his/her assessments. A larger α value indicates that the assessments are closer to the most possible value (with μ=1). For example, the initial cost of alternative B was assigned a triangular MF as a measure of the precision with which the cost is known to the expert.



a) Alternative A.



b) Alternative B.

Figure 2: Membership functions for the competing alternatives

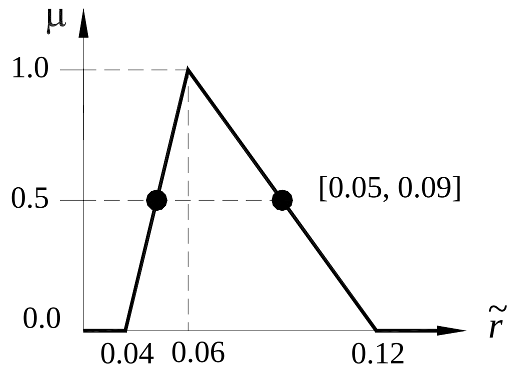


Figure 3: Membership function for the discount rate

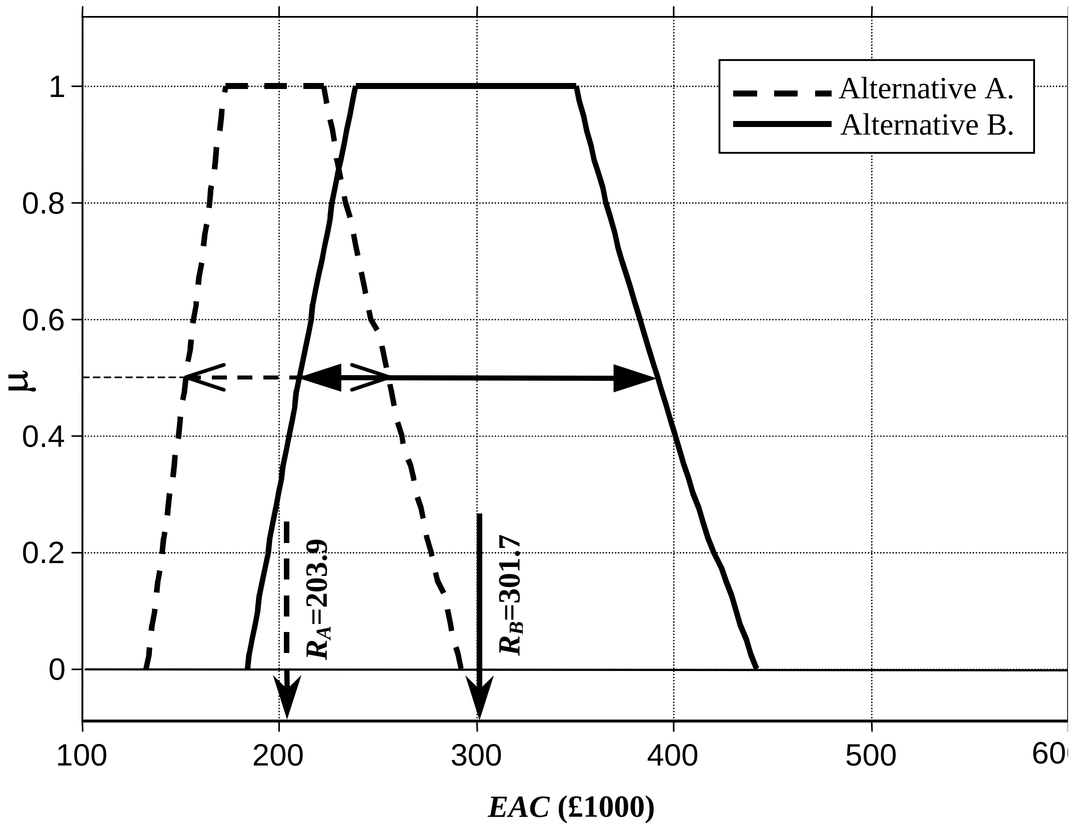


Figure 4: Membership functions of equivalent annual costs

Using 40 α-cuts, the proposed algorithm was employed to solve the example problem. Figure (4) depicts the resulting EACs of both alternatives. Alternative A seems to have a clear advantage over alternative B. Removals for both alternatives, R_A and R_B, are £210874 per year and £309439 per year, respectively.

Despite the relatively small number of employed α-cuts, smooth output MFs were obtained. This shows the robustness and computational efficiency of the algorithm. In this context, it is worth mentioned that to attain a smooth distribution using a conventional MCS would require thousands of simulations.

Another unique feature of the algorithm can be seen from figures (2-4). For any given α-cut, the range in the output variable, EAC_i, depends only on the corresponding ranges of the input variables. Thus, the impact of varying all input variables at a given α-level can be read off directly from the corresponding α-cut of the output function, EAC_i. This is illustrated on figures (2-4) for α=0.5. The 0.5-cuts for input variables are marked with solid dots in figures. (2 and 3). The corresponding output 0.5-cuts for alternatives A and B are depicted on figure (4) with open and solid arrowheads, respectively.

An α-cut may be interpreted as the range in which a given variable is judged to lie at a degree of precision defined by that level (Watson *et al.*, 1979). Thus, the proposed algorithm can be considered as a multivariable sensitivity analysis to which has been added a measure of the precision with which the input variables are known.

CONCLUSIONS AND FUTURE RESEARCH

A FST-based algorithm was designed specifically to permit the use of subjective judgements elicited from experts in LCC-based decision-making. The algorithm was designed around an explicit mathematical model. This model was formulated to introduce life cycle costs as an equivalent annual cost to allow the effective choice between alternatives with different lives.

Vital issues necessary for an effective implementation of the algorithm were carefully investigated. These issues include the employment of an efficient computer representation of fuzzy quantities, the optimisation of fuzzy operations and the choice of an effective ranking procedure. The proposed approach was demonstrated in the context of an example problem.

The inclusion of the analysis of uncertainty into the formulated LCC model is another unique feature of the algorithm. In addition, it was shown that the handling of uncertainty within the algorithms is superior to its treatment in conventional risk assessment techniques. Moreover, the algorithm has an efficient automatic ranking procedure. Therefore, the proposed algorithm has the potential to provide the decision-maker with a deeper insight into the problem at hand. This can put him/her in a better position to make an informed decision.

Last but not least, the algorithm can be extended to handle other quantitative and qualitative aspects of LCC. Considerable research effort aimed to building an LCC-based decision support system is being conducted at the school of Construction, Property and Surveying, the Robert Gordon University. The theoretical framework of this integrated system is outlined in Kishk and Al-Hajj (1999).

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APPENDIX: LIST OF SYMBOLS

$\tilde{A}$	A Symbols marked with a tilde represents a fuzzy quantity.
$\tilde{A}_{ij}$	Annual recurring costs of alternative i.
$A\tilde{E}I_i$	Annual equivalence factor for the initial cost, C_{0i} .
$A\tilde{E}N_{ik}$	Annual equivalence factor for a non-annual recurring cost, C_{ik} .
$A\tilde{E}O_{im}$	Annual equivalence factor for a future one-off cost, F_{im} .
$A\tilde{E}S_i$	Annual equivalence factor for the salvage value, S_i .
$[a,b]$	An interval where a ≤ b (the range of all possible values from a to b).
$\tilde{C}_{0i}$	Initial cost of alternative i.
$^a\tilde{C}_{0i}$	Annualised initial costs of alternative i.
$\tilde{C}_{ik}$	Non-annual recurring costs of alternative i.
$^aN\tilde{R}C_i$	Annualised non-annual recurring costs of alternative i.
f_{ik}	Frequencies of non-annual recurring costs, C_{ik} , of alternative i.
$\tilde{F}_{im}$	Single non-recurring (one-off) future costs of alternative i.
$^a\tilde{F}_i$	Annualised single one-off future costs of alternative i.
$\mathrm{int}(a)$	Rounds a to the nearest integer (towards zero).
$\tilde{n}_{ik}$	Number of recurrences of non-annual recurring cost, C_{ik} , of alternative i.
nar_i	Number of annual recurring costs.
noo_i	Number of future one-off (non-recurring) costs.
nnr_i	Number of non-annual recurring costs, C_{ik} .
$P\tilde{W}A_i$	Present worth factor of annual recurring costs.
$P\tilde{W}S_i$	Present worth factor for a single future cost.
$\tilde{r}$	Discount rate.
R	First ranking criterion (Removal).
$\mathrm{rem}\left(\frac{a}{b}\right)$	Remainder after division of two numbers a and b.
$\tilde{S}_i$	The salvage value of alternative i.
$^aS\tilde{A}V_i$	Annualised salvage value of alternative i.
$\tilde{t}_{im}$	Times of future one-off costs of alternative i.
$\tilde{T}_i$	The life of alternative i (in years).